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SPECIAL ASTRODYNAMIC REPORT NO. 2

LEVEL II

PROJECT SPACE TRACK

GENERAL PERTURBATIONS THEORIES DERIVED FROM THE 1965 LANE DRAG THEORY

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SPACETRACK REPORT NO. 2

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GENERAL PERTURBATIONS THEORIES DERIVED FROM THE 1965 LANE DRAG THEORY,

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The Air Force General Perturbation theory (AFGP4) based on the analytic satellite theory of Lane (1965) and Lane and Cranford (1969) which models the gravitational zonal harmonics through J_5 and models the atmosphere with a spherically symmetric power density function is given here in its complete form. All equations needed for satellite prediction are given and the reader is referred to the original publications for the theoretical background. Two simplified equation subsets (IGP4 and SGP4) of AFGP4 are also given and the procedure by which they were obtained is outlined. The IGP4 and SGP4 theories were originally developed by Kenneth H. Cranford.

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CONTENTS

1. INTRODUCTION

The analytic drag theories developed by the Office of Astrodynamic Applications are all descendants of a parent theory AFGP4. The lineage can be traced from AFGP4 to IGP4 and finally to SGP4. In general, the theories decrease in complexity and accuracy in the order AFGP4, IGP4, SGP4. This report gives a comprehensive description of the equations as they are implemented and describes the process by which the simplified theories were obtained.

2. AFGP4

AFGP4 is a general perturbations theory based on the work of Lane (1965) and Lane and Cranford (1969). The gravitational model includes the effects of the first five zonal harmonics of the Earth. The atmospheric model assumes a static, non-rotating, spherically symmetrical atmosphere whose density ρ can be described by the power function

$$\rho = \rho_o \left(\frac{q_o - s}{r - s}\right)^{\tau}$$

where the quantities on the right side are defined in the list of symbols in the Appendix. A discussion of the development of the equations is thoroughly covered in the publications cited above and will not be repeated here. The numerous symbols used in the following equations are defined in the Appendix.

Given the values at an epoch t_o of the mean orbital elements (denoted by subscript o) predictions with AFGP4 are made according to the following scheme:

a. Calculate the constants

$$k = B\mu(1-\eta^2)^{-7/2} \{(1 + \frac{3}{4} e^2 + \frac{3}{2} \eta^2 + 3e^2\eta^2 + 4e\eta + e\eta^3)$$

$$+ (\frac{3k_2}{2a}) (- \frac{1}{2} + \frac{3}{2} \theta^2)\xi(1-\eta^2)^{-1} [(8 + 24\eta^2 + 3\eta^4)$$

$$- 5e\eta(4 + 3\eta^2)]\}$$

$$A_1 = L^{-1}\xi^4$$

$$A_2 = 4\xi^9\mu^{-1}$$

$$A_3 = (4/3)\ L^{-1}(17a+s)\xi^{14}\mu^{-1}$$

$$A_4 = (2/3)\ (221a+31s)\xi^{19}\mu^{-2}$$

where all quantities on the right side are understood to be double-primed epoch quantities.

b. Calculate the time-dependent quantities

$$Q_1 = -(\frac{1}{3}) [k^2 \ (A_2+2A_1^2)\ (t-t_o)^3 + 0.25k^3\ (3A_3 + 12A_1A_2$$

$$+ 10A_1^3)\ (t-t_o)^4 + 0.20k^4\ (3A_4 + 12A_1A_3+6A_2^2 + 30A_1^2A_2$$

$$+ 15A_1^4)\ (t-t_o)^5]$$

$$Q_2 = -[A_2k^2(t-t_o)^2 + A_3k^3(t-t_o)^3 + A_4k^4(t-t_o)^4]$$

$$\ell_i'' = \ell_o'' + \{1 + \frac{3}{2}\frac{k_2}{a^2\beta^3}(-1+3\theta^2) + \frac{3}{32}\frac{k_2^2}{a^4\beta^7}[-15+16\beta+25\beta^2$$

$$+ (30-96\beta-90\beta^2)\theta^2 + (105+144\beta+25\beta^2)\theta^4] + \frac{15}{16}\frac{k_4}{a^4\beta^7}e^2(3-30\theta^2$$

$$+ 35\theta^4)\}\, n\,(t-t_o)$$

$$g_i'' = g_o'' + \{-\frac{3}{2}\frac{k_2}{a^2\beta^4}(1-5\theta^2) + \frac{3}{32}\frac{k_2^2}{a^4\beta^8}[-35 + 24\beta + 25\beta^2$$

$$+ (90-192\beta-126\beta^2)\theta^2 + (385+360\beta+45\beta^2)\theta^4] + \frac{5}{16}\frac{k_4}{a^4\beta^8}[21-9\beta^2$$

$$+ (-270+126\beta^2)\theta^2 + (385-189\beta^2)\theta^4]\}\, n\,(t-t_o)$$

$$h_i'' = h_o'' + \{-3\frac{k_2}{a^2\beta^4}\theta + \frac{3}{8}\frac{k_2^2}{a^4\beta^8}[(-5+12\beta+9\beta^2)\theta + (-35-36\beta-5\beta^2)\theta^3]$$

$$+ \frac{5}{4}\frac{k_4}{a^4\beta^8}\theta(3-7\theta^2)(5-3\beta^2)\}\, n\,(t-t_o)$$

where all quantities on the right side are understood to be double-primed epoch quantities.

c. From ℓ_i'', use the definitions of the angles to calculate E_i'' and λ_i''.

Then calculate

$$I_1 = \int_{t_0}^{t}\int_{\lambda_o''}^{\lambda_i''} (\gamma_2^{\tau-1} + 2e\gamma_1\gamma_2^{\tau-2} + \frac{3}{2} e^2\gamma_1^2\gamma_2^{\tau-3}$$

$$+ e^3\gamma_1^3\gamma_2^{\tau-4} + \frac{7}{8} e^4\gamma_1^4\gamma_2^{\tau-5})\, d\lambda\, dt$$

$$= \frac{1}{2\eta} \{ (1 + \frac{2e}{\eta} + \frac{3e^2}{2\eta^2} + \frac{e^3}{\eta^3} + \frac{7e^4}{8\eta^4}) J_{\tau-1} - (1-\eta^2) (\frac{2e}{\eta} + \frac{3e^2}{\eta^2}$$

$$+ \frac{3e^3}{\eta^3} + \frac{7e^4}{2\eta^4}) J_{\tau-2} + (1-\eta^2)^2 (\frac{3e^2}{2\eta^2} + \frac{3e^3}{\eta^3} + \frac{21e^4}{4\eta^4}) J_{\tau-3}$$

$$- (1-\eta^2)^3 (\frac{e^3}{\eta^3} + \frac{7e^4}{2\eta^4}) J_{\tau-4} + (1-\eta^2)^4 \frac{7e^4}{8\eta^4} J_{\tau-5} \} (\ell_i'' - \ell_o'')^2$$

$$I_2 = \int_{t_0}^{t}\int_{\lambda_o''}^{\lambda_i''} (\gamma_2^{\tau-1} - \frac{1}{2} e^2\gamma_1^2\gamma_2^{\tau-3})\, d\lambda\, dt$$

$$= \frac{1}{2\eta} \{ (1 - \frac{e^2}{2\eta^2}) J_{\tau-1} + \frac{e^2}{2\eta^2} (1-\eta^2) [2 J_{\tau-2}$$

$$- (1-\eta^2) J_{\tau-3}] \} (\ell_i'' - \ell_o'')^2$$

$$\delta \int_{t_0}^{t} \Delta L_i \, dt = - \frac{B\tau(\tau+1)\xi^{\tau+2}}{4an} [9k_2^2 (- \frac{1}{2} + \frac{3}{2}\theta^2)^2$$

$$+ \frac{a^2A_{3,0}^2\sin^2 I}{32k_2^2} + \frac{k_2^2\sin^4 I}{4}] \; (1-\eta^2)^{\tau+\frac{1}{2}} \; I_{o,\tau+2} \; (\ell_i'' - \ell_o'')^2$$

$$- \frac{2B\tau(\tau+1)\beta^8\xi^\tau a^3}{9n(1-5\theta^2)^2} \; \{- \frac{aA_{3,0}\sin I}{8k_2^2} \; (\sin^2 I+12) \; I_{1,\tau+2}[\sin g_i''$$

$$- \sin g_o'' - (g_i'' - g_o'') \cos g_o'')] + \frac{\sin^2 I}{2} \; [\frac{a^2A_{3,0}^2}{32k_2^4}$$

$$+ \frac{3}{4} (3\theta^2-1)] \; I_{2,\tau+2} \; [- \frac{1}{2} (\cos 2g_i'' - \cos 2g_o'') - (g_i'' -$$

$$- g_o'') \sin 2g_o''] + \frac{aA_{3,0}\sin^3 I}{24k_2^2} \; I_{3,\tau+2} \; [\frac{1}{3} (\sin 3g_i'' - \sin 3g_o'')$$

$$- (g_i'' - g_o'') \cos 3g_o''] + \frac{\sin^4 I}{16} \; I_{4,\tau+2}[-\frac{1}{4} (\cos 4g_i'' - \cos 4g_o'')$$

$$-(g_i'' - g_o'') \sin 4g_o''] \}$$

$$\int_{t_o}^{t} \Delta L_i dt = - B\alpha a \; [1 - \frac{3k_2}{a^2} \left(\frac{L}{G}\right)^3 (-\frac{1}{2} + \frac{3}{2}\theta^2)] \; I_1$$

$$- Bk_2\xi^\tau (-\frac{1}{2} + \frac{3}{2}\theta^2) \; \frac{1}{2n}[1 - \frac{3k_2}{a^2} \left(\frac{L}{G}\right)^3 (-\frac{1}{2} + \frac{3}{2}\theta^2)] \; (\ell_i''$$

$$- \ell_o'')^2 \; \{-\tau\xi[-3I_{o,\tau+1} - \frac{9}{2} e \; I_{1,\tau+1} + e^2 \; (-\frac{9}{2} I_{o,\tau+1} - I_{2,\tau+1})$$

$$+ e^3 \; (- \frac{97}{16} I_{1,\tau+1}$$

$$- \frac{9}{16} I_{5,\tau+1})] + \frac{3}{a} [I_{o,\tau} + \frac{11}{2} eI_{1,\tau} + e^2 (4I_{o,\tau}$$

$$+ 5I_{2,\tau})\} + \frac{a^4\beta^8 B\xi^\tau (1-\theta^2)}{3nk_2(1-5\theta^2)^2} \{-\tau\xi[\frac{1}{6} I_{2,\tau+1}$$

$$- \frac{e}{3} I_{3,\tau+1} + e^2 (- \frac{1}{16} I_{o,\tau+1} + \frac{1}{8} I_{2,\tau+1}$$

$$+ \frac{17}{48} I_{4,\tau+1}) + \frac{e^3}{24} (-3I_{1,\tau+1} + 8I_{3,\tau+1} + 7I_{5,\tau+2})]$$

$$+ \frac{1}{a} [I_{2,\tau} + \frac{e}{4} (3I_{1,\tau} + 11 I_{3,\tau}) + \frac{e^2}{4} (I_{o,\tau}$$

$$+ 8I_{2,\tau} + 17I_{4,\tau})]\} [\cos 2g_i'' - \cos 2g_o''$$

$$+ 2(g_i'' - g_o'') \sin 2g_o''] + \frac{a^4\beta^6 B\xi^\tau}{3nk_2(1-5\theta^2)^2} \{\frac{1}{24} [1$$

$$- 11\theta^2 - \frac{40\theta^4}{1-5\theta^2}] - \frac{5}{36} \frac{k_4}{k_2^2} [1 - 3\theta^2$$

$$- \frac{8\theta^4}{1-5\theta^2}]\} \{-\tau\xi[-eI_{1,\tau+1} + \frac{e^2}{2} (-I_{o,\tau+1} - 3I_{2,\tau+1})$$

$$+ \frac{e^3}{8} (-3I_{1,\tau+1} - 9I_{3,\tau+1})] + \frac{3}{a} [eI_{1,\tau}$$

$$+ \frac{e^2}{2} (I_{o,\tau} + 3I_{2,\tau})]\} \times [\cos 2g_i'' - \cos 2g_o''$$

$$+ 2\,(g_i'' - g_o'')\sin 2g_o'']$$

$$+ \frac{a^5 \beta^8 B \xi^\tau A_{3,o} \sin I}{9k_2^3 n(1-5\theta^2)^2}\{-\tau\xi[-I_{1,\tau+1} + \frac{e}{2}(-I_{o,\tau+1}$$

$$- 3I_{2,\tau+1}) + \frac{e^2}{8}(-3I_{1,\tau+1} - 9I_{3,\tau+1})$$

$$+ \frac{e^3}{16}(3I_{o,\tau+1} - 8I_{2,\tau+1} - 11\,I_{4,\tau+1})]$$

$$+ \frac{3}{a}[I_{1,\tau} + \frac{e}{2}(I_{o,\tau} + 3I_{2,\tau}) + \frac{e^2}{8}(9I_{1,\tau}$$

$$+ 11\,I_{3,\tau})]\}\,[\sin g_i'' - \sin g_o''$$

$$- (g_i'' - g_o'')\cos g_o''\,] + \delta\int_{t_o}^{t}\Delta L_i\,dt + Q_1$$

$$\int_{t_o}^{t}\Delta G_i\,dt = \int_{t_o}^{t}\Delta H_i\,dt = -\,B\alpha a[1 - \frac{3k_2}{a^2}\left(\frac{L}{G}\right)^3(-\frac{1}{2}$$

$$+ \frac{3}{2}\theta^2)]\,I_2 - \frac{3Bk_2\xi^\tau}{2n}(-\frac{1}{2} + \frac{3}{2}\theta^2)\tau\xi I_{o,\tau+1}(\ell_i''$$

$$- \ell_o'')^2 - \frac{Ba^4\xi^\tau\beta^8}{18nk_2}\,\frac{(1-\theta^2)\tau\xi}{(1-5\theta^2)^2}\{\cos 2g_i''$$

$$- \cos 2g_o'' + 2(g_i'' - g_o'')\sin 2g_o''\}$$

$$- \frac{Ba^4 \xi^\tau \beta^8 \tau \xi}{3nk_2 (1-5\theta^2)^2} \{ \frac{e}{\beta^2} \frac{1}{24} [1 - 11\theta^2 - \frac{40\theta^4}{1-5\theta^2}]$$

$$- \frac{5}{36} \frac{k_4}{k_2^2} [1 - 3\theta^2 - \frac{8\theta^4}{(1-5\theta^2)}]\} I_{1,\tau+1} \quad \{\cos 2g''_i$$

$$- \cos 2g''_o + 2(g''_i - g''_o) \sin 2g''_o \}$$

$$+ \frac{Ba^5 \xi^\tau \beta^8 \tau \xi A_{3,0}}{9nk_2^3 (1-5\theta^2)^2} (\sin I) \; I_{1,\tau+1} \quad \{\sin g''_i$$

$$- \sin g''_o - (g''_i - g''_o) \cos g''_o \} + Q_1$$

where all quantities on the right side (except ℓ''_i, λ''_i and g''_i) are understood to be double-primed epoch quantities.

d. Then, calculate the next approximation for ℓ'', g'', and h'' from

$$\ell''_{i+1} = \ell''_i - 3n[1 + \frac{2k_2}{a^2 \beta^3} (-1 + 3\theta^2)] \int_{t_o}^{t} \Delta L_i \, dt$$

$$- \frac{9}{2} n \frac{k_2}{a^2 \beta^3} (-1+5\theta^2) \int_{t_o}^{t} \Delta G_i \, dt$$

$$+ 9n \frac{k_2}{a^2 \beta^3} \theta^2 \int_{t_o}^{t} \Delta H_i \, dt$$

$$g''_{i+1} = g''_i + \frac{9}{2} n \frac{k_2}{a^2 \beta^4} (1-5\theta^2) \int_{t_o}^{t} \Delta L_i \, dt$$

$$+ \; 3n \frac{k_2}{a^2\beta^4} (2-15\theta^2) \int_{t_o}^{t} \Delta G_i \; dt$$

$$+ \; 15n \frac{k_2}{a^2\beta^4} \theta^2 \int_{t_o}^{t} \Delta H_i \; dt$$

$$h''_{i+1} = h''_i + 9n \frac{k_2\theta}{a^2\beta^4} \int_{t_o}^{t} \Delta L_i \; dt$$

$$+ \; 15n \frac{k_2\theta}{a^2\beta^4} \int_{t_o}^{t} \Delta G_i \; dt + 3n \frac{k_2\theta}{a^2\beta^4} \int_{t_o}^{t} \Delta H_i \; dt$$

where all quantities on the right side are double-primed epoch quantities unless otherwise noted. Repeat step c, using ℓ''_{i+1}, g''_{i+1}, h''_{i+1} in place of ℓ''_i, g''_i, h''_i, respectively. The number of iterations required is determined by the value of $X = \left| n \int \Delta L_i \; dt \right|$ according to the following table:

X	Iterations
$.5 \leq X$	4
$.05 \leq X < .5$	2
$X < .05$	1

e. Let ℓ''_s, g''_s, h''_s denote the final iterated values, and let λ''_s and E''_s denote the values of λ'' and E'' corresponding to ℓ''_s. Then calculate

$$I_3 = \int_{\lambda_o''}^{\lambda_s''} \sin\lambda \quad (2\gamma_2{}^{\tau-2} + 2e\gamma_1\gamma_2{}^{\tau-3})d\lambda = -\frac{2}{\eta^2} \quad \{\frac{\eta+e}{\tau-1} [(1$$

$$+ \eta \cos \lambda_s'')^{\tau-1} - (1 + \eta \cos \lambda_o'')^{\tau-1}] - \frac{e(1-\eta^2)}{\tau-2} [(1$$

$$+ \eta \cos \lambda_s'')^{\tau-2} - (1 + \eta \cos \lambda_o'')^{\tau-2}]\}$$

$$I_4 = -\frac{1}{\eta(\tau-1)} [(1 + \eta \cos \lambda_s'')^{\tau-1} - (1 + \eta \cos \lambda_o'')^{\tau-1}]$$

$$\frac{1}{L} F_2 = -\frac{B\tau(\tau+1)\xi^{\tau+2}}{2a} [9k_2^2 (-\frac{1}{2} + \frac{3}{2}\theta^2)^2 + \frac{a^2A_{3,0}^2}{32k_2^2} \sin^2 I$$

$$+ \frac{k_2^2 \sin^4 I}{4}] \int_{\lambda_o}^{\lambda_s''} \gamma_2{}^{\tau+1} d\lambda \quad + \frac{B\tau(\tau+1)\beta^4\xi^{\tau+2}a}{3(1-5\theta^2)} \quad \times$$

$$\{-\frac{aA_{3,0}\sin I}{8k_2} (\sin^2 I + 12) \; I_{1,\tau+2}[\cos g_s'' - \cos g_o'']$$

$$+ \frac{\sin^2 I}{2} [\frac{a^2A_{3,0}^2}{32k_2^3} + \frac{3k_2}{4}(3\theta^2 - 1)] I_{2,\tau+2} \; [\sin 2g_s'' - \sin 2g_o'']$$

$$+ \frac{aA_{3,0}\sin^3 I}{24k_2} I_{3,\tau+2} [\cos 3g''_s - \cos 3g''_o]$$

$$+ \frac{k_2 \sin^4 I}{16} I_{4,\tau+2} [\sin 4g''_s - \sin 4g''_o]\}$$

$$L'' = L''_o \quad [1 - B\alpha a[1 - \frac{3k_2}{a^2} \frac{L^3}{G^3} (- \frac{1}{2} + \frac{3}{2} \theta^2)] \int_{\lambda''_o}^{\lambda''_s} (\gamma_2{}^{\tau-1}$$

$$+ 2e\gamma_1\gamma_2{}^{\tau-2}$$

$$+ \frac{3}{2} e^2\gamma_1^2\gamma_2{}^{\tau-3})d\lambda - Bk_2\xi^\tau (- \frac{1}{2} + \frac{3}{2} \theta^2) (\ell''_s$$

$$-\ell''_o) \{ -\tau\xi[-3I_{o,\tau+1} - \frac{9}{2} eI_{1,\tau+1}]$$

$$+ \frac{3}{a} I_{o,\tau}\} + Ba^2\xi^\tau\beta^4 \frac{(1-\theta^2)}{(1-5\theta^2)} \{-\tau\xi[\frac{1}{6} I_{2,\tau+1}$$

$$+ \frac{1}{3} eI_{3,\tau+1}]$$

$$+ \frac{1}{a} I_{2,\tau}\} (\sin 2g''_s - \sin 2g''_o)$$

$$+ \frac{Ba^2\xi^\tau\beta^4}{(1-5\theta^2)} \{\frac{e}{\beta^2} (\frac{1}{24} [1-11\theta^2 - \frac{40\theta^4}{1-5\theta^2}]$$

$$- \frac{5}{36} \frac{k_4}{k_2^2} [1-3\theta^2 - \frac{8\theta^4}{1-5\theta^2}]) \ (\sin 2g_s'' - \sin 2g_s'')$$

$$- \frac{1}{6} \frac{A_{3,0}}{k_2^2} a \ \sin I \ (\cos g_s'' - \cos g_o'')\} \ \{-\tau\xi[-I_{1,\tau+1}$$

$$- \frac{1}{2} e \ (I_{o,\tau+1} + 3I_{2,\tau+1})] + \frac{3}{a} I_{1,\tau}\} + \frac{1}{L} F_2] + L_o'' Q_2$$

$$H = H_o'' \ [1-B\alpha a[1 - \frac{3k_2}{a^2} \left(\frac{L}{G}\right)^3 (-\frac{1}{2} + \frac{3}{2} \theta^2)] \int_{\lambda_o''}^{\lambda_s''} (\gamma_2^{\tau+1}$$

$$- \frac{1}{2} e^2 \gamma_1^2 \gamma_2^{\tau-3}) d\lambda - BK_2 \xi^\tau \ (-\frac{1}{2} + \frac{3}{2} \theta^2) \ (\ell_s''$$

$$- \ell_o'') \ \{-\tau\xi \ [- \ 3I_{o,\tau+1} + \frac{3}{2} eI_{1,\tau+1}] + \frac{1}{a} [3I_{o,\tau}]\}$$

$$+ \frac{Ba^2 \xi^\tau \beta^4 (1-\theta^2)}{1-5\Theta^2} \ \{-\tau\zeta \ [\frac{1}{6} I_{2,\tau+1} - \frac{e}{6} (I_{1,\tau+1} - I_{3,\tau+1})]$$

$$+ \frac{1}{3a} I_{2,\tau}\} [\sin 2g_s'' - \sin 2g_o'']$$

$$+ \frac{Ba^2\xi^\tau\beta^4}{1-5\theta^2}\{\frac{e}{\beta^2}(\frac{1}{24}[1-11\theta^2 - \frac{40\theta^4}{1-5\theta^2}]$$

$$- \frac{5}{36}\frac{k_4}{k_2^2}[1-3\theta^2 - \frac{8\theta^4}{1-5\theta^2}])(\sin 2g''_s - \sin 2g''_o)$$

$$- \frac{aA_{3,0}}{6k_2^2}\sin I\ (\cos g''_s - \cos g''_o)\}\{-\tau\xi\ \ [-I_{1,\tau+1}$$

$$+ \frac{e}{2}(I_{o,\tau+1} - I_{2,\tau+1})]$$

$$+ \frac{1}{a} I_{1,\tau}\} + \frac{1}{L} F_2\ ] + H\ Q_2.$$

$$e''_D = e''_o - B\alpha a\beta^2 \int_{\lambda''_o}^{\lambda''_s}[2\gamma_1\gamma_2{}^{\tau-2} + 2e\gamma_1{}^2\gamma_2{}^{\tau-3}]d\lambda$$

$$-B\beta^2\xi^\tau k_2\ (-\frac{1}{2} + \frac{3}{2}\theta^2)\ (E''_s - E''_o)\ \{\tau\ \xi[6I_{1,\tau+1}$$

$$+ \frac{3}{2} e\ (I_{o,\tau+1} + I_{2,\tau+1})] + \frac{15}{a} I_{1,\tau}\}$$

$$+ \frac{1}{2} B\beta^5 a^2\xi^\tau \frac{1-\theta^2}{1-5\theta^2}\{\tau\xi[-\frac{4}{3} I_{1,\tau+1} - \frac{2}{3} I_{3,\tau+1}$$

$$+ e\ (\frac{1}{3} I_{o,\tau+1} + \frac{1}{3} I_{2,\tau+1} - \frac{1}{3} I_{4,\tau+1})] + \frac{1}{a}[I_{1,\tau}$$

$$+ \frac{7}{3} I_{3,\tau}]\}[\sin 2g''_s - \sin 2g''_o]$$

$$+ \frac{B\beta^4 a^2 \xi^\tau}{3(1-5\theta^2)} \left(\frac{1}{8} [1-11\theta^2 - \frac{40\theta^4}{1-5\theta^2}] \right.$$

$$- \frac{5}{12} \frac{k_4}{k_2^2} [1-3\theta^2 - \frac{8\theta^4}{1-5\theta^2}]) \{\tau\xi [eI_{o,\tau+1} + eI_{2,\tau+1}]$$

$$- \frac{2}{a} I_{1,\tau}\} \quad [\sin 2g''_s - \sin 2g''_o]$$

$$- \frac{B\beta^6 a^3 \xi^\tau A_{3,0} \sin I}{6(1-5\theta^2)k_2^2} \quad \{\tau\xi[I_{o,\tau+1} + I_{2,\tau+1}$$

$$+ e \ (I_{1,\tau+1} \quad I_{3,\tau+1})] + \frac{3}{a} (I_{o,\tau+1})\}[\cos g''_s$$

$$- \cos g''_o]$$

and the drag periodics

$$e''_o \delta\ell_D = B\alpha a\sqrt{1-\eta^2} \ I_3 + Ba^2\xi^\tau\beta^5 \frac{1-\theta^2}{1-5\theta^2} \{- \frac{1}{6} \tau\xi[I_{3,\tau+1}$$

$$- I_{1,\tau+1} + e \ (\frac{1}{4} I_{o,\tau+1} - \frac{25}{8} I_{2,\tau+1} + \frac{15}{4} I_{4,\tau+1}$$

$$- \frac{7}{8} I_{6,\tau+1})] + \frac{1}{6a} [-3I_{1,\tau} \quad + 7I_{3,\tau}] \} \ [\cos 2g''_s$$

$$- \cos 2g''_o \ ] + \frac{Ba\xi^\tau\beta^3}{1-5\theta^2} \{ \frac{1}{24} (1-11\theta^2 - \frac{40\theta^4}{1-5\theta^2})$$

$$- \frac{5}{36} \frac{k_4}{k_2^2} (1-3\theta^2 - \frac{8\theta^4}{1-5\theta^2})\} \quad [-\tau a\xi e(I_{o,\tau+1}$$

$$- I_{2,\tau+1}) + 2I_{1,\tau}] \; [\cos 2g_s'' - \cos 2g_o'']$$

$$- \frac{Ba^2\xi^\tau\beta^5 A_{3,o} \sin I}{6k_2^2 \; (1-5\theta^2)} \; \{\tau a\xi \; [I_{o,\tau+1} - I_{2,\tau+1}$$

$$+ e \; (I_{1,\tau+1} - I_{3,\tau+1})] + 3 \; (I_{o,\tau} - I_{2,\tau})\} \; [\sin g_s''$$

$$- \sin g_o'']$$

$$\delta I_D = \frac{Ba\xi^\tau\beta^4\theta}{1-5\theta^2} \; \{\frac{2}{3} \; I_{2,\tau} \; \sin I \; [\sin 2g_s'' - \sin 2g_o'']$$

$$- \frac{aA_{3,0}}{3k_2^2} \; I_{1,\tau} \; [\cos g_s'' - \cos g_o'']\}$$

$$(\sin \frac{I_D''}{2}) \; \delta h_D = - \frac{2}{3} \frac{Ba\xi^\tau\theta}{(1-5\theta^2)} \, (\sin \frac{I}{2}) \; I_{2,\tau} \; [\cos 2g_s'' - \cos 2g_o'']$$

$$- \frac{Ba^2\xi^\tau\beta^2 A_{3,0}\theta}{6k_2^2 \; (1-5\theta^2) \; \cos \frac{I}{2}} \; [I_{1,\tau} + \frac{e}{2} \; (- I_{o,\tau}$$

$$+ I_{2,\tau})] \; [\sin g_s'' - \sin g_o'']$$

$$\delta e_D = e''_D - e''_o$$

where all quantities on the right side (except g''_s, E''_s and λ''_s) are understood to be double-primed epoch quantities and where

$$\int \gamma_1{}^p \gamma_2^m d\gamma = \sum_{k=0}^{p} \sum_{\ell=0}^{m} \eta^{p-k+\ell} \binom{p}{k} \binom{m}{\ell} \Big\{ \frac{1}{k+\ell} \cos^{k+\ell-1}\lambda \sin\lambda$$

$$+ \sum_{j=2}^{i-1} \frac{\prod_{k=2}^{j} (\ell-k+3)}{\prod_{k=1}^{j} (\ell-k+2)} \cos^{k+\ell-2j+1}\lambda \sin\lambda + R_i \Big\},$$

$$\text{with } i = \begin{cases} \dfrac{k+\ell+1}{2} & \text{if } k+\ell \text{ is odd} \\ \dfrac{k+\ell+2}{2} & \text{if } k+\ell \text{ is even} \end{cases}$$

$$\text{and } R_i = \begin{cases} \lambda & \text{if } k+\ell=0 \\ 0 & \text{if } k+\ell=1 \\ \prod_{k=2}^{i} \left(\dfrac{2k-3}{2k-2}\right)\lambda & \text{if } k+\ell \text{ is even and } k+\ell \geq 2 \\ \prod_{k=2}^{i} \left(\dfrac{2k-3}{2k-1}\right) \sin\lambda & \text{if } k+\ell \text{ is odd and } k+\ell \geq 3 \end{cases}$$

Then the inclination due to the drag, I''_D, can be calculated from

$$H'' = L'' \sqrt{1-e''^2_D} \quad \cos I''_D$$

f. The drag periodic effects on the angle variables and eccentricity are now included by first calculating

$$\sin \frac{I''}{2} \cos h'' = \cos h''_s \left[\frac{\delta I_D}{2} \quad \cos \frac{I''_D}{2} + \sin \frac{I''_D}{2} \right]$$

$$- \sin h''_s \sin (I''_D/2) \; \delta h_D$$

$$\sin \frac{I''}{2} \sin h'' = \sin h''_s \left[\frac{\delta I_D}{2} \quad \cos \frac{I''_D}{2} + \sin \frac{I''_D}{2} \right]$$

$$+ \cos h''_s \sin \left(\frac{I''_D}{2}\right) \delta h_D$$

$$e'' \cos \ell'' = (e''_o + \delta e_D) \cos \ell''_s - e''_o \; \delta \ell_D \sin \ell''_s$$

$$e'' \sin \ell'' = (e''_o + \delta e_D) \sin \ell''_s + e''_o \; \delta \ell_D \cos \ell''_s$$

$$\ell'' + g'' + h'' = \ell''_s + g''_s + h''_s$$

Then, the complete set of double-primed elements at time t is

$$e'' = \sqrt{(e'' \sin \ell'')^2 + (e'' \cos \ell'')^2}$$

$$\ell'' = \tan^{-1} [e'' \sin \ell'' / e'' \cos \ell'']$$

$$h'' = \tan^{-1} [(\sin I''/2 \sin h'') / (\sin I''/2 \cos h'')]$$

$$g'' = (\ell'' + g'' + h'') - \ell'' - h''$$

$$I'' = 2 \sin^{-1} \sqrt{(\sin I''/2 \sin h'')^2 + (\sin I''/2 \cos h'')^2}$$

and L" has already been given in step e.

g. The periodics due to the geopotential model are now included by first calculating

$$\ell + g + h = (\ell'' + g'' + h'') + \frac{k_2}{4a^2\beta^2} (\frac{e}{1+\beta}) \{2(-1+3\theta^2) (\frac{a^2\beta^2}{r^2}$$

$$+ \frac{a}{r} + 1) \sin f + 3(1-\theta^2) [(- \frac{a^2\beta^2}{r^2} - \frac{a}{r} +$$

$$+ 1) \sin (2g + f) + (\frac{a^2\beta^2}{r^2} + \frac{a}{r} + \frac{1}{3}) \sin (2g + 3f)]\}$$

$$+ \frac{3}{2} \frac{k_2}{a^2\beta^4} (-1-2\theta+5\theta^2) \quad (f - \ell + e \sin f)$$

$$+ \frac{k_2}{4a^2\beta^4} (3+2\theta-5\theta^2) [3 \sin (2g + 2f)$$

$$+ 3e \sin (2g + f) + e \sin (2g + x)]$$

$$+ \frac{1}{16} \frac{k_2}{a^2\beta^4} \{(2\beta^3-2-e^2) - 11 (2\beta^3-2-3e^2)\theta^2$$

$$- 40(2\beta^3-2-5e^2)\theta^4 \quad (1-5\theta^2)^{-1} + 400\ e^2\theta^6\ (1-5\theta^2)^{-2}$$

$$- 2e^2\theta[11 + 80\theta^2\ (1-5\theta^2)^{-1}$$

$$+ 200\theta^4\ (1-5\theta^2)^{-2}]\}\ \sin 2g - \frac{5}{24}\ \frac{k_4}{k_2 a^2 \beta^4}\ \{(2\beta^3$$

$$-2-e^2) - 3\ (2\beta^3-2-3e^2)\theta^2 - 8\ (2\beta^3-2-5e^2)\ \frac{\theta^4}{1-5\theta^2}$$

$$+ 80e^2\theta^6\ (1-5\theta^2)^{-2}\ -2e^2\ \theta[3 + 16\theta^2\ (1-5\theta^2)^{-1}$$

$$+ 40\theta^4\ (1-5\theta^2)^{-2}]\}\ \ \sin 2g + \frac{1}{4}\ \frac{A_{30}}{k_2 a \beta^2}\ [(1+\beta$$

$$+\beta^2)\ \ (\frac{e}{1+\beta})\ \sin I + e\theta\ \tan I/2]\ \cos g + \frac{5}{64}$$

$$+ \frac{A_{50}}{k_2 a^3 \beta^6}\ [4\beta^2\ \sin I\ (\frac{e}{1+\beta}) + 3e\beta^2\ \sin I - 9e\beta^3\ \sin I$$

$$+ (4 + 3e^2)e\ \ \theta \tan I/2 + e \sin I\ (26$$

$$+ 9e^2)]\ \ [1-9\theta^2 - 24\theta^4\ (1-5\theta^2)^{-1}]\ \cos g$$

$$+ \frac{15}{32}\ \frac{A_{50}}{k_2 a^3 \beta^6}\ e\theta\ \sin I\ (1-\theta)\ (4 + 3e^2)\ [3 + 16\theta^2\ (1$$

$$-5\theta^2)^{-1} + 40\theta^4 \ (1-5\theta^2)^{-2}] \cos g$$

$$+ \frac{35}{1152} \frac{A_{50}}{k_2 a^3 \beta^6} [e \sin I \ (3\beta^3 - 3 - 2e^2)$$

$$-e^3 \ \theta \tan I/2] \ [1-5\theta^2 - 16\theta^4 \ (1-5\theta^2)^{-1}] \cos 3g$$

$$- \frac{35}{576} \frac{A_{50}}{k_2 a^3 \beta^6} e^3 \ \theta \sin I \ (1-\theta) \ [5+32\theta^2 \ (1-5\theta^2)^{-1}$$

$$+ 80\theta^4 \ (1-5\theta^2)^{-2}] \cos 3g$$

$$L = L'' \{1 + \frac{k_2}{2a^2} [3(\frac{a}{r})^3 \ (1-\theta^2) \cos (2g + 2f) + (3\theta^2$$

$$-1) \ ((\frac{a}{r})^3 - \beta^{-3})]\}$$

$$e''\delta\ell = - \frac{k_2}{4a^2\beta} 2(-1+3\theta^2) \ (\frac{a^2\beta^2}{r^2} + \frac{a}{r} + 1) \sin f$$

$$+ 3(1-\theta^2) \ [(- \frac{a^2\beta^2}{r^2} - \frac{a}{r} + 1) \sin (2g + f)$$

$$+(\frac{a^2\beta^2}{r^2} + \frac{a}{r} + \frac{1}{3}) \sin (2g + 3f)]\}$$

$$+\{\frac{k_2 e}{8a^2} \ [1-11\theta^2 - 40\theta^4 \ (1-5\theta^2)^{-1}]$$

$$- \frac{5k_4 e}{12k_2 a^2} [1-3\theta^2 - 8\theta^4 (1-5\theta^2)^{-1}]\} \sin 2g + - \frac{1}{4} \frac{A_{3,0}\beta}{k_2 a} \sin I$$

$$- \frac{5}{64} \frac{A_{5,0}}{k_2 a^3 \beta^3} \sin I (4 + 9e^2) [1 - 9\theta^2 - 24\theta^4 (1$$

$$-5\theta^2)^{-1}]\} \cos g + \frac{35}{384} \frac{A_{5,0}}{k_2 a^3 \beta^3} e^2 \sin I [1-5\theta^2-16\theta^2 (1$$

$$-5\theta^2)^{-1}] \cos 3g$$

$$\delta e = \delta_1 e + \frac{k_2}{2a^2\beta^4} \{(-1+3\theta^2) [e\beta + \frac{e}{1+\beta} + 3 \cos f + 3e \cos^2 f$$

$$+ e^2 \cos^3 f] + 3(1-\theta^2) [e + 3 \cos f + 3e \cos^2 f$$

$$+ e^2 \cos^3 f] \cos (2g + 2f)\} - \frac{k_2}{2\beta^2 a^2} (1-\theta^2) [3 \cos (2g$$

$$+f) + \cos (2g + 3f)]$$

where $\delta_1 e = \sin I \, \delta_2 e$, where

$$\delta_2 e = \{\frac{1}{8} \frac{k_2 e}{a^2\beta^2} [\frac{1-15\theta^2}{1-5\theta^2} \sin I] - \frac{5k_4 e}{12k_2 a^2 \beta^2} [\frac{1-7\theta^2}{1-5\theta^2} \sin I]\} \cos 2g$$

$$+ \{\frac{1}{4} \frac{A_{3,0}}{k_2 a} + \frac{5}{64} \frac{A_{5,0}}{k_2 a^3 \beta^4} (4 + 3e^2)[1-9\theta^2-24\theta^4(1-5\theta^2)^{-1}]\} \sin g$$

$$- \frac{35}{384} (\frac{A_{5,0}}{k_2 a^3 \beta^4}) e^2 [1-5\theta^2-16\theta^4 (1-5\theta^2)^{-1}] \sin 3g$$

$$\delta I = -\frac{e\theta\delta_2 e}{\beta^2} + \frac{k_2\ \theta \sin I}{2a^2\beta^4}\ [3\cos(2g + 2f) + 3e\cos(2g$$

$$+ f) + e\cos(2g + 3f)]$$

$$(\sin I''/2)\delta h = -\ (\frac{k_2}{2a^2\beta^4})\ (\sin I/2)\ [6(f - \ell + e\sin f)$$

$$- 3\sin(2g + 2f) - 3e\sin(2g + f)$$

$$- e\sin(2g + 3f)] + \sin I/2\ \ -\{\frac{1}{8}\ \frac{k_2 e^2}{a^2\beta^4}\ \theta[11$$

$$+ 80\theta^2\ (1-5\theta^2)^{-1} + 200\theta^4\ (1-5\theta^2)^{-2}]$$

$$+ \frac{5}{12}\ \ (\frac{k_4}{k_2 a^2\beta^4})\ e^2\ \theta[3 + 16\theta^2\ (1-5\theta^2)^{-1}$$

$$+ 40\theta^4\ (1-5\theta^2)^{-2}]\}\ \sin 2g + \{\frac{1}{4}\ \frac{A_{30}}{k_2 a\beta^2}\ \ (\frac{e\theta}{2\cos I/2})$$

$$+ \frac{5}{64}\ \frac{A_{50}}{k_2 a^3\beta^6}\ \ (\frac{e\theta}{2\cos I/2})\ (4 + 3e^2)\ [1-9\theta^2$$

$$- 24\theta^4\ (1-5\theta^2)^{-1}]$$

$$+ \frac{15}{32} \frac{A_{50}}{k_2 a^3 \beta^6} e\ \theta \sin I/2 \sin I\ (4 + 3e^2)\ [3$$

$$+ 16\theta^2 (1-5\theta^2)^{-1} + 40\theta^4 (1-5\theta^2)^{-2}]\}\ \cos g$$

$$+ \{- \frac{35}{1152} \frac{A_{50}}{k_2 a^3 \beta^6} \left(\frac{e^3 \theta}{2 \cos I/2} \right) [1-5\theta^2$$

$$- 16\theta^4 (1-5\theta^2)^{-1}]$$

$$- \frac{35}{576} \frac{A_{50}}{k_2 a^3 \beta^6} e^3\ \theta \sin I \sin I/2\ [5 + 32\theta^2 (1$$

$$-5\theta^2)^{-1} + 80\theta^4 (1-5\theta^2)^{-2}]\}\ \cos 3g$$

$H = H''$

where all quantities on the right side are understood to be double-primed elements at time t.

h. The osculating orbital elements can now be determined by first calculating

$$\sin I/2 \cos h = \cos h'' [(\delta I/2) \cos I''/2 + \sin I''/2]$$

$$- \sin h'' \sin (I''/2)\ \delta h$$

$$\cos I/2 \sin h = \sin h'' \left[\delta I/2 \;\; \cos I''/2 + \sin I''/2\right]$$

$$+ \cos h'' \sin (I''/2) \quad \delta h$$

$$e \cos \ell = (e'' + \delta e) \cos \ell'' - e'' \delta\ell \sin \ell''$$

$$e \sin \ell = (e'' + \delta e) \sin \ell'' + e'' \delta\ell \cos \ell''$$

$$\ell + g + h = \ell'' + g'' + h''$$

Then, the complete set of osculating elements at time t is

$$e = \sqrt{(e \sin \ell)^2 + (e \cos \ell)^2}$$

$$\ell = \tan^{-1} \left[e \sin \ell / e \cos \ell\right]$$

$$h = \tan^{-1} \left[(\sin I/2 \sin h) / (\sin I/2 \cos h)\right]$$

$$g = (\ell + g + h) - \ell - h$$

$$I = 2 \sin^{-1} \sqrt{(\sin I/2 \sin h)^2 + (\sin I/2 \cos h)^2}$$

$$a = L^2/\mu$$

3. Drag Simplification

The drag portion of AFGP4 is simplified by using only the secular terms from the equations along with certain drag periodics of important magnitude. The value of τ is fixed at 4 throughout the simplified equations, and all quantities are understood to be mean epoch elements unless otherwise noted.

a. The L" equation from AFGP4 (section 2e.) is approximated by

$$L'' = L''_o \{1 - B\alpha a \int_{\lambda''_o}^{\lambda''_s} (\gamma_2{}^3 + 2e\gamma_1\gamma_2{}^2 + 3/2\ e^2\gamma_1{}^2\gamma_2)d\lambda$$

$$- Bk_2\xi^4\ (-1/2 + 3/2\theta^2)\ (\ell''_s - \ell''_o)\ 12\xi\ I_{o,5}\}$$

$$+ L''_o\ Q_2.$$

Upon evaluating the integrals, we have the secular part

$$L'' = L''_o \{1 - B\psi^{-7}\xi^4 a\ (1 + 3/2\eta^2 + 4e\eta + e\eta^3 + 3/4e^2$$

$$+ 3e^2\eta^2)\ (\lambda''_s - \lambda''_o) - 3/2\ Bk_2\xi^4\ (-1/2$$

$$+ 3/2\theta^2)\ [\xi\psi^{-9}\ (8 + 24\eta^2 + 3\eta^4)]\ (\ell''_s - \ell''_o)\}$$

$$+ L''_o\ Q_2$$

If we assume that

$$\lambda''_s - \lambda''_o = \ell''_s - \ell''_o = n''_o (t - t_o)$$

and drop certain smaller terms, we have

$$L'' = L''_o [1 - C_1 (t - t_o)] + L''_o Q_2$$

where

$$C_1 = B\xi^4 n \psi^{-7} [a (1 + \tfrac{3}{2}\eta^2 + 4e\eta + e\eta^3) + \tfrac{3}{2} k_2 \xi \psi^{-2} (- \tfrac{1}{2}$$

$$+ \tfrac{3}{2}\theta^2) (8 + 24\eta^2 + 3\eta^4)]$$

Dropping the same small terms, the constant k (section 2a.) is approximately by

$$k = na^2 C_1 \xi^{-4}$$

It then follows that Q_2 (section 2b.) can be written

$$Q_2 = - D_2 (t - t_o)^2 - D_3 (t - t_o)^3 - D_4 (t - t_o)^4$$

where

$$D_2 = 4a \xi C_1{}^2$$

$$D_3 = \tfrac{4}{3} a\xi^2 (17a + s) C_1{}^3$$

$$D_4 = \tfrac{2}{3} a^2 \xi^3 (221a + 31s) C_1{}^4$$

Then

$$L'' = L''_o\,[1 - C_1\,(t - t_o) - D_2\,(t - t_o)^2 - D_3\,(t - t_o)^3$$

$$- D_4\,(t - t_o)^4]$$

b. From section 2c., we make the approximation

$$\int_{t_o}^{t} \Delta L_i\,dt = -B\alpha a\,I_1 - Bk_2\xi^4\,(-\tfrac{1}{2} + \tfrac{3}{2}\theta^2)\,\frac{1}{2n}\,(\ell''_i$$

$$- \ell''_o)^2\,12\xi I_{o,5} + Q_1$$

Using the same approximation as in section 3a., we take

$$\int_{t_o}^{t} \Delta L_i\,dt = -\tfrac{1}{2}\,C_1\,(t - t_o)^2 + Q_1$$

where

$$Q_1 = -\tfrac{1}{3}\,[(D_2 + 2\,C_1^{\,2})\,(t - t_o)^3 + \tfrac{1}{4}\,(3D_3 + 12\,C_1\,D_2$$

$$+ 10\,C_1^{\,3})\,(t - t_o)^4 + \tfrac{1}{5}\,(3D_4 + 12\,C_1\,D_3 + 6D_2^{\,2}$$

$$+ 30\,C_1^{\,2}\,D_2 + 15\,C_1^{\,4})\,(t - t_o)^5]$$

It is further assumed that

$$\int_{t_o}^{t} \Delta G_i \, dt = \int_{t_o}^{t} \Delta H_i \, dt = -\frac{1}{2} C_1 (t - t_o)^2$$

An iteration is not performed on the equations in section 2d. so that with the above approximations, we have

$$\ell''_s = \ell''_i + n \left[\frac{3}{2} C_1 (t - t_o)^2 + (D_2 + 2 C_1^2)(t - t_o)^3 + \frac{1}{4}(3D_3 + 12 C_1 D_2 + 10 C_1^3)(t - t_o)^4 + \frac{1}{5}(3D_4 + 12 C_1 D_3 + 6D_2^2 + 30 C_1^2 D_2 + 15 C_1^4)(t - t_o)^5\right]$$

$$g''_s = g''_i$$

$$h''_s = h''_i - \frac{21}{2} \frac{n k_2 \theta}{a^2 \beta^2} C_1 (t - t_o)^2$$

where the effects of ΔG_i and ΔH_i on ℓ''_s and g''_s have been ignored.

c. The drag periodic $e''_o \, \delta\ell_D$ from section 2e. is approximated by

$$e''_o \, \delta\ell_D = B\alpha a \quad \psi I_3 - \frac{B a^2 \xi^4 \beta^5 A_{3,0} \sin I}{6 k_2^2 (1 - 5\theta^2)} (4 a \xi I_{o,5}) (\sin g''_s - \sin g''_o)$$

The integral I_3 is approximated by

$$I_3 = -\frac{2}{3}\frac{1}{\eta}\left[(1+\eta\cos\ell_i'')^3 - (1-\eta\cos\ell_o'')^3\right]$$

so that

$$e_o''\,\delta\ell_D = -\frac{2}{3}\,Ba\xi^4\psi^{-6}\,\frac{1}{\eta}\left[(1+\eta\cos\ell_i'')^3 - (1-\eta\cos\ell_o'')^3\right]$$

$$+\;Ba\beta n\xi^5\psi^{-9}\,\frac{A_{3,0}}{k_2}\,\sin I\,(1+3\eta^2+\tfrac{3}{8}\eta^4)\cos g\,(t-t_o)$$

where the approximation

$$\sin g_s'' - \sin g_o'' = (t-t_o)\frac{d}{dt}(\sin g_s'')$$

$$= -\frac{3}{2}\,\frac{k_2 n}{a^2\beta^4}\,(1-5\theta^2)\sin g\,(t-t_o)$$

has been made. Finally with further approximations, we take

$$\delta\ell_D = -\frac{2}{3}\,B\xi^4\,\frac{a}{e\eta}\left[(1+\eta\cos\ell_i'')^3 - (1+\eta\cos\ell_o'')^3\right]$$

$$+\;B^*\,C_3\cos g\,(t-t_o)$$

where

$$C_3 = (q_o - s)^4\,\xi^5\,an\,\frac{A_{3,0}}{k_2 e}\sin I$$

The drag period δg_D is taken to be

$$\delta g_D = -\delta \ell_D$$

d. The e''_D equation from AFGP4 (section 2e.) is approximated by

$$e''_D = e''_o - B\alpha a\beta^2 \int_{\lambda''_o}^{\lambda''} (2\gamma_1\gamma_2{}^2 + 2e\gamma_1\gamma^2\gamma_2)\, d\lambda$$

$$- B\beta^2\xi^5 k_2 \left(-\frac{1}{2} + \frac{3}{2}\theta^2\right) (E''_s - E''_o) \Big[24\, I_{1,5}$$

$$+ 6e\, (I_{o,5} + I_{2,5}) + \frac{15}{a\xi} I_{1,4}\Big]$$

$$+ 2\, B\beta^6 a^2 \xi^5 \frac{(1-\theta^2)}{(1-5\theta^2)} \Big[-\frac{4}{3} I_{1,5} - \frac{2}{3} I_{3,5}$$

$$+ e\left(\frac{2}{3} I_{o,5} + \frac{1}{3} I_{2,5} - \frac{1}{3} I_{4,5}\right) + \frac{1}{2a\xi} \Big(I_{1,4}$$

$$+ \frac{7}{3} I_{3,4}\Big)\Big] (\sin 2g''_s - \sin 2g''_o)$$

The integral term, when evaluated, is

$$- 2\, B\psi^{-7}\xi^4\, a\beta^2 \left(2\eta + \frac{1}{2}\eta^3 + \frac{1}{2} e + 2e\eta^2\right) (\lambda'' - \lambda''_o)$$

$$- 2\ B\psi^{-7}\xi^{4}a\beta^{2}\ [(1 + \frac{11}{4}\eta^{2} + \frac{11}{4}e\eta + e\eta^{3})\ (\sin\lambda''$$

$$- \sin\lambda''_{o}) + (\frac{1}{2}\eta + \frac{1}{4}\eta^{3} + \frac{1}{4}e + \frac{1}{2}e\eta^{2})\ (\sin 2\lambda''$$

$$- \sin 2\lambda''_{o}) + (\frac{1}{12}\eta^{2} + \frac{1}{12}e\eta)\ (\sin 3\lambda'' - \sin 3\lambda''_{o})]$$

which is approximated by

$$- 2\ B\psi^{-7}\xi^{4}a\beta^{2}n\ (2\eta + \frac{1}{2}\eta^{3} + \frac{1}{2}e + 2e\eta^{2})\ (t - t_{o})$$

$$- 2\ B\psi^{-7}\xi^{4}a\beta^{2}\ (1 + \frac{11}{4}\eta^{2} + \frac{11}{4}e\eta + e\eta^{3})\ (\sin\ell''$$

$$- \sin\ell''_{o})$$

where $\ell'' = \ell''_{s} + \delta\ell_{D}$. The second term gives

$$4\ B\beta^{2}\xi^{5}\psi^{-9}k_{2}n\ (1-3\theta^{2})\ (\frac{15}{2}\eta + \frac{45}{8}\eta^{3} + \frac{9}{2}e + \frac{9}{4}e\eta^{2} - \frac{3}{16}e\eta^{4})\ (t - t_{o})$$

where the approximation

$$E_s'' - E_o'' = n_o'' \quad (t - t_o)$$

has been made.

The third term given

$$2B\beta^2 \xi^5 \psi^{-9} k_2 n \ (1-\theta^2) \ (10\eta + \frac{65}{4}\eta^3 - \frac{7}{3} e - 13e\eta^2$$

$$+ \frac{31}{4} e\eta^4) \cos 2g$$

where the approximation

$$\sin 2g_s'' - \sin 2g_o'' = (t - t_o) \frac{d}{dt} (\sin 2g_s'')$$

$$= - 3 \frac{k_2 n}{a^2 \beta^4} \ (1-5\theta^2) \cos 2g \ (t - t_o)$$

has been made. Thus, the final equation for e_D'' becomes

$$e_D'' = e_o'' - B^* C_4 \ (t - t_o) - B^* C_5 \ [\sin (\ell_s'' + \delta\ell_D)$$

$$- \sin \ell_o'']$$

where†

$$C_4 = 2 \ (q_o - s)^4 \xi^4 \psi^{-7} \ a\beta^2 n \ \{[2\eta + \frac{1}{2}\eta^3 + \frac{1}{2}e + 2e\eta^2]$$

---

† It should be noted that the k_2 terms of C_4 in the operational version do not agree with this equation. The present equation has been verified as correct with the author and theoperational version will be modified to agree.

$$- 2 \frac{k_2 \xi}{a} \psi^{-2} [(1-3\theta^2) \left(\frac{15}{2}\eta + \frac{45}{8}\eta^3 + \frac{9}{2}e + \frac{9}{4}e\eta^2 \right.$$

$$\left. - \frac{3}{16}e\eta^4\right) + (1-\theta^2) \left(5\eta + \frac{65}{8}\eta^3 - \frac{7}{6}e - \frac{13}{2}e\eta^2\right.$$

$$\left. + \frac{31}{8}e\eta^4\right) \cos 2g] \}$$

$$C_5 = 2 \, (q_o - s)^4 \, \xi^4 \psi^{-7} a \beta^2 \, (1 + \frac{11}{4}\eta^2 + \frac{11}{4}e\eta + e\eta^3)$$

e. The double-prime updated elements are

$$e'' = e''_D$$

$$I'' = I''_o$$

$$\ell'' = \ell''_s + \delta\ell_D$$

$$g'' = g_s + \delta g_D$$

$$h'' = h''_s$$

and L'' has been given in section 3a.

4. IGP4

IGP4 is obtained from AFGP4 by using the drag simplification discussed in section 3 while retaining the full Brouwer geopotential (1959) with the Lyddane modification (1963).

Given the values at an epoch t_o of the mean orbital elements (denoted by subscript o) predictions with IGP4 are made according to the following scheme:

a. Calculate the constants

$$C_2 = (q_o - s)^4 \xi^4 n \, (1 - \eta^2)^{-9/2} \, [a \, (1 - \eta^2) \, (1 + \tfrac{3}{2}\eta^2$$

$$4e\eta + e\eta^3) + \tfrac{3}{2} k_2 \xi \, (-\tfrac{1}{2} + \tfrac{3}{2}\theta^2) \, (8 + 24\eta^2 + 3\eta^4)]$$

$$C_1 = B^* C_2$$

$$C_3 = \frac{(q_o - s)^4 \xi^5 A_{3,0} n \; a \sin I}{k_2 e}$$

$$C_4 = 2n \, (q_o - s)^4 \xi^4 a \beta^2 \, (1 - \eta^2)^{-7/2} \Big([2\eta \, (1 + e\eta)$$

$$+ \tfrac{1}{2}e + \tfrac{1}{2}\eta^3] - \frac{2k_2\xi}{a \, (1 - \eta^2)} \, [(1-3\theta^2) \, (\tfrac{15}{2}\eta + \tfrac{45}{8}\eta^3$$

$$+ \frac{9}{2e} + \tfrac{9}{4}e\eta^2 - \tfrac{3}{16}e\eta^4) + (1-\theta^2) \, (5\eta + \tfrac{65}{8}\eta^3 - \tfrac{7}{6}c$$

$$- \frac{13}{2} e\eta^2 + \frac{31}{8} e\eta^4) \cos 2g] \Big)$$

$$C_5 = 2 (q_o - s)^4 \xi^4 a\beta^2 (1 - \eta^2)^{-7/2} [1 + \frac{11}{4}\eta (\eta + e)$$

$$+ e\eta^3]$$

$$D_2 = 4a \xi C_1^{\,2}$$

$$D_3 = \frac{4}{3} a \xi^2 (17a + s) C_1^{\,3}$$

$$D_4 = \frac{2}{3} a^2 \xi^3 (221a + 31s) C_1^{\,4}$$

where all quantities on the right side are understood to be double-primed epoch quantities.

b. Calculate the time dependent quantities

$$\ell_i'' = \ell_o'' + \{1 + \frac{3}{2} \frac{k_2}{a^2\beta^3} (-1+3\theta^2) + \frac{3}{32} \frac{k_2^2}{a^4\beta^7} [- 15 + 16\beta$$

$$+ 25\beta^2 + (30 - 96\beta - 90\beta^2)\theta^2 + (105 + 144\beta$$

$$+ 25\beta^2)\theta^4] + \frac{15}{16} \frac{k_4}{a^4\beta^7} e^2 (3-30\theta^2 + 35\theta^4)\} n(t - t_o)$$

$$g_i'' = g_o'' + \{- \frac{3}{2} \frac{k_2}{a^2\beta^4} (1-5\theta^2) + \frac{3}{32} \frac{k_2^2}{a^4\beta^8} [- 35 + 24\beta$$

$$+ 25\beta^2 + (90 - 192\beta - 126\beta^2)\theta^2 + (385 + 360\beta$$

$$+ 45\beta^2)\theta^4] + \frac{5}{16}\frac{k_4}{a^4\beta^8}[21 - 9\beta^2 + (-270 + 126\beta^2)\theta^2$$

$$+ (385 - 189\beta^2)\theta^4]\} \; n(t - t_o)$$

$$h''_i = h''_o + \{-3\frac{k_2}{a^2\beta^4}\theta + \frac{3}{8}\frac{k_2^2}{a^4\beta^8}[(-5 + 12\beta + 9\beta^2)\theta$$

$$+ (-35 - 36\beta - 5\beta^2)\theta^3] + \frac{5}{4}\frac{k_4}{a^4\beta^8}\theta\,(3-7\theta^2)\,(5$$

$$- 3\beta^2)\} \; n(t - t_o)$$

where all quantities on the right side are understood to be double-primed epoch quantities.

c. Let ℓ''_s, g''_s, h''_s denote the secular values of ℓ'', g'', h'', respectively. Then

$$g''_s = g''_i$$

$$h''_s = h''_i - \frac{21}{2}\frac{nk_2\theta}{a^2\beta^2}C_1\,(t - t_o)^2$$

$$\ell_s'' = \ell_i'' + n\,[\tfrac{3}{2}\,C_1\,(t - t_o)^2 + (D_2 + 2C_1{}^2)\,(t - t_o)^3$$

$$+ \tfrac{1}{4}\,(3D_3 + 12C_1D_2 + 10C_1{}^3)\,(t - t_o)^4 + \tfrac{1}{5}\,(3D_4$$

$$+ 12C_1D_3 + 6D_2{}^2 + 30C_1{}^2D_2 + 15C_1{}^4)\,(t - t_o)^5]$$

Also

$$L'' = L_o''\,[1 - C_1\,(t - t_o) - D_2\,(t - t_o)^2 - D_3\,(t - t_o)^3$$

$$- D_4\,(t - t_o)^4]$$

$$e_D'' = e_o'' - B^*C_4\,(t - t_o) - B^*C_5\,[\sin\,(\ell_s'' + \delta\ell_D)$$

$$- \sin\,\ell_o'']$$

$$I_D'' = I_o''$$

where all quantities on the right side are understood to be double-primed epoch quantities.

d. Calculate the drag periodics

$$\delta\ell_D = -\,\frac{2}{3}\,(q_o - s)^4 B^*\xi^4\,\frac{a}{e\eta}\,\,[(1 + \eta\,\cos\,\ell_i'')^3$$

$$-\,(1 + \eta\,\cos\,\ell_o'')^3] + B^*C_3\,(\cos\,g_o'')\,(t - t_o)$$

$$\delta g_D = -\delta \ell_D$$

where all quantities on the right side (except ℓ_i'') are understood to be double-primed epoch quantities.

e. The complete set of double-primed elements at time t is

$$e'' = e_D''$$

$$I'' = i_D''$$

$$\ell'' = \ell_s'' + \delta \ell_D$$

$$g'' = g_s'' + \delta g_D$$

$$h'' = h_s''$$

and L" has been given in section 4c.

f. The geopotential transformation for IGP4 osculating orbital elements is then given by the equations of section 2g and 2h.

5. Geopotential Simplification

The geopotential portion of AFGP4 is simplified by assuming eccentricity is small and then retaining only the main terms. The small eccentricity problem is handled by using an alternate set of variables. The small inclination problem is not treated since few satellites which experience drag forces have small inclinations.

a. The secular geopotential equations (section 2b.) are simplified by taking $\beta=1$ except for those β appearing in the denominators. The result is

$$\ell_i'' = \ell_o'' + [1 + \frac{3k_2(1- + 3\theta^2)}{2a^2\beta^3}$$

$$+ \frac{3k_2^2\ (13 - 78\theta^2 + 137\theta^4)}{16a^4\beta^7}]\ n(t - t_o)$$

$$g_i'' = g_o'' + [- \frac{3k_2\ (1 - 5\theta^2)}{2a^2\beta^4} + \frac{3k_2^2\ (7 - 114\theta^2 + 395\theta^4)}{16a^4\beta^8}$$

$$+ \frac{5k_4\ (3 - 36\theta^2 + 49\theta^4)}{4a^4\beta^8}]\ n(t - t_o)$$

$$h_i'' = h_o'' + [- \frac{3k_2\theta}{a^2\beta^4} + \frac{3k_2^2\ (4 - 19\theta^3)}{2a^4\beta^8}$$

$$+ \frac{5k_4\theta\ (3 - 7\theta^2)}{2a^4\beta^8}\ ]\ n(t - t_o)$$

where all quantities on the right side are understood to be double-primed epoch quantities.

b. The long-period terms of the geopotential (section 2g.) are approximated by retaining only the main $A_{3,0}$ term. The result is

$$\delta(\ell'' + g'' + h'')_L + \frac{1}{8} \frac{A_{3,0}}{k_2 a\beta^2} e \sin I \left(\frac{3+5\theta}{1+\theta}\right) \cos g$$

$$\delta L_L = 0$$

$$\delta I_L = 0$$

$$\delta h_L = 0$$

$$\delta e_L = \frac{1}{4} \frac{A_{3,0}}{k_2 a\beta^2} \sin I \sin g$$

$$\delta g_L = \frac{1}{4} \frac{A_{3,0}}{k_2 a\beta^2} \frac{1}{e} \sin I \cos g$$

where all quantities on the right side are understood to be double-primed elements at time t, and where the subscript L denotes long-period geopotential periodics.

If we consider the two new variables $e' \sin g'$ and $e' \cos g'$, then

$$e' \sin g' = e'' \sin g'' + \delta(e'' \sin g'')_L$$

$$= e'' \sin g'' + \delta e_L \sin g'' + e'' \delta g_L \cos g''$$

$$e' \cos g' = e'' \cos g'' + \delta(e'' \cos g'')_L$$

$$= e'' \cos g'' + \delta e_L \cos g'' - e'' \delta g_L \sin g''$$

Upon substituting, we find

$$\delta(e'' \sin g'') = \frac{1}{4} \frac{A_{3,0}}{k_2 a \beta^2} \sin I$$

$$\delta(e'' \cos g'') = 0$$

Then the long-period geopotential transformation is

$$\ell' + g' + h' = \ell'' + g'' + h'' + \delta(\ell'' + g'' + h'')_L$$

$$e' \sin g' = e'' \sin g'' + \delta(e'' \sin g'')_L$$

$$e' \cos g' = e'' \cos g''$$

$$L' = L''$$

$$I' = I''$$

$$h' = h''$$

and it follows that

$$e' = \sqrt{(e' \sin g')^2 + (e' \cos g')^2}$$

c. Before adding the short-period terms, a change of variables is made according to the following scheme. Let E' denote the eccentric anomaly corresponding to ℓ'. Solve Kepler's equation for E' + g' (by iteration to the desired accuracy) where

$$(E' + g')_{i+1} = (E' + g')_i + \Delta(E' + g')_i$$

with

$$\Delta(E' + g')_i =$$

$$\frac{\ell'+g' - (e' \sin g') \cos (E'+g')_i + (e' \cos g') \sin (E'+g')_i - (E'+g')_i}{-(e' \sin g') \sin (E' + g')_i - (e' \cos g') \cos (E'+g')_i + 1}$$

and

$$(E' + g')_1 = \ell' + g'$$

Then, compute the following

$$e' \cos E' = (e' \cos g') \cos (E' + g')$$
$$+ (e' \sin g') \sin (E' + g')$$

$$e' \sin E' = (e' \cos g') \sin (E' + g')$$
$$- (e' \sin g') \cos (E' + g')$$

$$p' = a' (1 - e'^2)$$

where $a' = \frac{1}{\mu} L'^2$

and

$$r' = a' (1 - e' \cos E')$$

$$\dot{r}' = \frac{L'}{r'} e' \sin E'$$

$$r' \dot{f}' = \frac{L' \sqrt{1 - e'^2}}{r'}$$

where f' denotes the true anomaly corresponding to ℓ'.

Let $u' = f' + g'$. Then

$$\cos u' = \frac{a'}{r'} [\cos (E' + g') - e' \cos g'$$

$$+ \frac{(e' \sin g') (e' \sin E')}{1 + \sqrt{1 - e'^2}}]$$

$$\sin u' = \frac{a'}{r'} [\sin (E' + g') - e' \sin g'$$

$$- \frac{(e' \cos g') (e' \sin E')}{1 + \sqrt{1-e'^2}}]$$

At this point we have the following set of single-primed variables.

$$r', \dot{r}', r'\dot{f}', I', h', u'$$

d. For the above set of single-primed variables, we can write the following formulas.

$$(\delta r)_s = \frac{ae}{\beta} \sin f \, (\delta \ell)_s + \frac{r}{a} (\delta a)_s - a \cos f \, (\delta e)_s$$

$$(\delta \dot{r})_s = - \frac{1}{2} \frac{ne}{\beta} \sin f \, (\delta a)_s + \frac{na}{\beta} \left(\frac{a}{r}\right) (1+e \cos f) \sin f \, (\delta e)_s$$

$$+ nae \cos f \left(\frac{a}{r}\right)^2 (\delta \ell)_s$$

$$\delta(r\dot{f})_s = -\frac{1}{2} n\beta \left(\frac{a}{r}\right) (\delta a)_s + \frac{na}{\beta} \left(\frac{a}{r}\right) (\cos f - e$$

$$+ e \cos^2 f) (\delta e)_s - nae \sin f \left(\frac{a}{r}\right)^2 (\delta \ell)_s$$

$$(\delta u)_s = \frac{1}{\beta^2} (2 + e \cos f) \sin f (\delta e)_s + \frac{1}{\beta^3} (1$$

$$+ e \cos f)^2 (\delta \ell)_s + (\delta g)_s$$

where all quantities on the right side are single-primed quantities and when the subscript s denotes short-period geopotential periodics. We then substitute from the Brouwer formulas, simplify, and make the approximations

$$(\delta r)_s = \frac{1}{2} \frac{k_2}{a^2\beta^3} (1-\theta^2) \cos 2u - \frac{3}{2} \frac{k_2}{a^2\beta^3} (3\theta^2-1) r$$

$$(\delta \dot{r})_s = 0$$

$$\delta(r\dot{f})_s = 0$$

$$\delta I = \frac{3}{2} \frac{k_2}{a^2\beta^4} \theta \sin I \cos 2u$$

$$\delta h = \frac{3}{2} \frac{k_2}{a^2\beta^4} \theta \sin 2u$$

$$\delta u = -\frac{1}{4}\frac{k_2}{a^2\beta^4}(7\theta^2-1)\sin 2u$$

where all quantities on the right side are single-primed quantities.

e. Osculating variables are given by

$$r = r' + (\delta r)_s$$

$$\dot{r} = \dot{r}'$$

$$r\dot{f} = r'\dot{f}'$$

$$I = I' + (\delta I)_s$$

$$h = h' + (\delta h)_s$$

$$u = u' + (\delta u)_s$$

and provide a very convenient set of variables for calculating position and velocity in Cartesian coordinates.

6. SGP4

SGP4 is obtained from IGP4 by using the geopotential simplification discussed in section 5.

Given the values at an epoch t_o of the mean orbital elements (denoted by subscript o) predictions with SGP4 are made according to the following scheme:

a. The drag and secular portions of the equations are given by following the equations of section 4a through 4e with the single exception that the secular geopotential effect of section 4b should be replaced by the shortened secular geopotential effect given in section 5a.

b. The long-period geopotential periodics are then added to the double-primed elements at time t in the following manner.

Compute the long-period gravitational periodics

$$\delta(\ell'' + g'' + h'') = \frac{A_{3,0} \sin I}{8k_2 a\beta^2} (e \cos g) \left(\frac{3 + 5\theta}{1 + \theta}\right)$$

$$\delta(e'' \sin g'') = \frac{A_{3,0} \sin I}{4k_2 a\beta^2}$$

where all quantities on the right side are understood to be double-primed elements at time t.

Then

$$\ell' + g' + h' = \ell'' + g'' + h'' + \delta(\ell'' + g'' + h'')$$

$$e' \sin g' = e'' \sin g'' + \delta(e'' \sin g'')$$

$$e' \cos g' = e'' \cos g''$$

It follows that

$$e' = \sqrt{(e' \sin g')^2 + (e' \cos g')^2}$$

Let E' denote the eccentric anomaly corresponding to ℓ'. Solve Kepler's equation for E' + g' (by iteration to the desired accuracy) where

$$(E' + g')_{i+1} = (E' + g')_i + \Delta(E' + g')_i$$

with

$$\Delta(E' + g')_i =$$

$$\frac{\ell' + g' - (e' \sin g') \cos (E'+g')_i + (e' \cos g') \sin (E'+g')_i - (E'+g')_i}{- (e' \sin g') \sin (E'+g')_i - (e' \cos g') \cos (E'+g')_i + 1}$$

and

$$(E' + g')_1 = \ell' + g'$$

Then, compute the following

$$e' \cos E' = (e' \cos g') \cos (E' + g') + (e' \sin g') \sin (E' + g')$$

$$e' \sin E' = (e' \cos g') \sin (E' + g')$$

$$- (e' \sin g') \cos (E' + g')$$

$$p' = a' (1 - e'^2)$$

where $a' = \frac{1}{\mu} L'^2$

and

$$r' = a' (1 - e' \cos E')$$

$$\dot{r}' = \frac{L'}{r'} e' \sin E'$$

$$r'\dot{f}' = \frac{L' \sqrt{1-e'^2}}{r'}$$

where f' denotes the true anomaly corresponding to ℓ'. Let $u' = f'+g'$. Then

$$\cos u' = \frac{a'}{r'} [\cos (E'+g') - e' \cos g'$$

$$+ \frac{(e' \sin g') (e' \sin E')}{1 + \sqrt{1 - e'^2}}$$

$$\sin u' = \frac{a'}{r'} \left[\sin (E' + g') - e' \sin g' - \frac{(e' \cos g' (e' \sin E')}{1 + \sqrt{1 - e'^2}}\right]$$

At this point we have the following set of single-primed variables

$$r', \dot{r}', r'\dot{f}', I', h', u'$$

c. Compute the short-period gravitational periodics

$$\delta r = \frac{k_2}{2p} (1-\theta^2) \cos 2u - \frac{3}{2} \frac{k_2}{a^2\beta^3} (3\theta^2-1)r$$

$$\delta u = - \frac{k_2}{4p^2} (7\theta^2-1) \sin 2u$$

$$\delta h = \frac{3k_2\theta}{2p^2} \sin 2u$$

$$\delta I = \frac{3k_2\theta}{2p^2} \sin I \cos 2u$$

where all quantities on the right side are understood to be single-primed variables at time t.

d. The osculating variables are now given by

$$r = r' + \delta r$$

$$u = u' + \delta u$$

$$h = h' + \delta h$$

$$I = I' + \delta I$$

$$\dot{r} = \dot{r}'$$

$$r\dot{f} = r'\dot{f}'$$

where all quantities on the right side are understood to be single-primed variables.

References

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Lane, M.: 1965, AIAA Paper No. 65-35.

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Appendix

a = semimajor axis

e = eccentricity

I = inclination

μ = product of Newton's gravitational constant and the mass of the Earth

$L = \sqrt{\mu a}$

$G = L\sqrt{1-e^2}$

$H = G \cos I$

ℓ = mean anomaly

g = argument of perigee

h = longitude of ascending node

a'' = "mean" semimajor axis

e'' = "mean" eccentricity

I'' = "mean" inclination

$L'' = \sqrt{\mu a''}$

$G'' = L''\sqrt{1 - e''^2}$

$H'' = G'' \cos I''$

ℓ'' = "mean" mean anomaly

g'' = "mean" argument of perigee

h'' = "mean" longitude of ascending node

ℓ''_s = secular value of ℓ''

g''_s = secular value of g''

h''_s = secular value of h''

q_o = geocentric reference altitude

ρ_o = atmospheric density at q_o

s = fitting parameter in density representation

τ = fitting parameter in density representation (restricted to integral values)

E'' = "mean" eccentric anomaly and has the same functional relation to e'' and ℓ'' as the eccentric anomaly E has to e and ℓ

f'' = "mean" true anomaly and has the same functional relation to e'' and ℓ'' as the true anomaly f has to e and ℓ

$$\eta'' = \frac{a'' e''}{a''-s}$$

$$\cos \lambda'' = \frac{\cos E'' - \eta''}{1-\eta'' \cos E''}$$

$$\sin \lambda'' = \frac{\sqrt{1-\eta''^2} \sin E''}{1-\eta'' \cos E''}$$

$$\gamma_1 = \eta'' + \cos \lambda''$$

$$\gamma_2 = 1 + \eta'' \cos \lambda''$$

$$\beta'' = \sqrt{1 - e''^2}$$

$$\xi'' = \frac{1}{a''-s}$$

$$\alpha = (1 - \eta''^2)^{1/2-\tau} \xi''^{\tau}$$

C_D = aerodynamic drag coefficient

A = effective cross-sectional area of satellite

m = mass of satellite

$$B_o = \frac{1}{2} C_D \frac{A}{m} \rho_o \left(\frac{qo - s}{a'' - s}\right)^{\tau}$$

$$B = B_o (a'' - s)^{\tau}$$

$$\theta'' = \cos I''$$

$n'' = \sqrt{\mu} \, (a'')^{-3/2}$ = "mean" mean motion

$k_2 = \frac{1}{2} J_2 R^2$ where R is the Earth's average equatorial radius and J_2 is the second zonal coefficient in the geopotential

$A_{3,0} = - J_3 R^3$ where J_3 is the third zonal coefficient in the geopotential

$k_4 = - \frac{3}{8} J_4 R^4$ where J_4 is the forth zonal coefficient in the geopotential

$A_{5,0} = - J_5 R^5$ where J_5 is the fifth zonal coefficient in the geopotential

$$I_{j,k} = \frac{1}{2\pi} \int_0^{2\pi} \frac{\cos jx}{(1 - \eta'' \cos x)^k} dx$$

$$= \frac{(\eta'')^j (1+\psi)^{-j}}{(k-1)!} f_k$$

where $\psi = \sqrt{1 - \eta''^2}$

$$f_1 = \psi^{-1}$$

$$f_2 = j\psi^{-2} + \psi^{-3}$$

$$f_3 = (j^2 - 1)\psi^{-3} + 3j\psi^{-4} + 3\psi^{-5}$$

$$f_4 = (j^3 - 4j)\psi^{-4} + (6j^2 - 9)\psi^{-5} + 15j\psi^{-6} + 15\psi^{-7}$$

$$f_5 = (j^4 - 10j^2 + 9)\psi^{-5} + (10j^3 - 55j)\psi^{-6}$$

$$+ (45j^2 - 90)\psi^{-7} + 105j\psi^{-8} + 105\psi^{-9}$$

$$f_6 = (j^5 - 20j^3 + 64j)\psi^{-6} + (15j^4 - 195j^2 + 225)\psi^{-7}$$

$$+ (105j^3 - 735j)\psi^{-8} + (420j^2 - 1050)\psi^{-9}$$

$$+ 945j\psi^{-10} + 945\psi^{-11}$$

$$f_7 = (j^6 - 35j^4 + 259j^2 - 225)\psi^{-7} + (21j^5 - 525j^3$$

$$+ 2079j)\psi^{-8} + (210j^4 - 3360j^2 + 4725)\psi^{-9}$$

$$+ (1260j^3 - 10{,}710j)\psi^{-10} + (4725j^2 - 14{,}175)\psi^{-11}$$

$$+ 10{,}395j\psi^{-12} + 10{,}395\psi^{-13}$$

$$f_8 = (j^7 - 56j^5 + 784j^3 - 2304j)\psi^{-8} + (28j^6 - 1190j^4 + 10{,}612j^2 - 11{,}025)\psi^{-9} + (378j^5 - 11{,}340j^3 + 53{,}487j)\psi^{-10} + (3150j^4 - 59{,}850j^2 + 99{,}225)\psi^{-11} + (17{,}325j^3 - 173{,}250j)\psi^{-12} + (62{,}370j^2 - 218{,}295)\psi^{-13} + 135{,}135j\psi^{-14} + 135{,}135\psi^{-15}$$

$$f_9 = (j^8 - 84j^6 + 1974j^4 - 12916j^2 + 11025)\psi^{-9} + (36j^7 - 2394j^5 + 39564j^3 - 136431j)\psi^{-10} + (630j^6 - 31500j^4 + 328545j^2 - 396400)\psi^{-11} + (6430j^5 - 242550j^3 + 1327095j)\psi^{-12} + (51975j^4 - 1143450j^2 + 2182950)\psi^{-13} + (270270j^3 - 3108105j)\psi^{-14} + (945945j^2 - 3783780)\psi^{-15} + 2027025j\psi^{-16} + 2027025\psi^{-17}$$

$$J_n = \frac{1}{2\pi} \int_o^{2\pi} (1 + \eta'' \cos x)^n \, dx$$

$$= \psi^{2n+1} I_{o,n+1}, \text{ for } n \geq o$$

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The Air Force General Perturbation theory (AFGP4) based on the analytic satellite theory of Lane (1965) and Lane and Cranford (1969) which models the gravitational zonal harmonics through J_5 and models the atmosphere with a spherically symmetric power density function is given here in its complete form. All equations needed for satellite prediction are given and the reader is referred to the original publications for theoretical (contd)

background. Two simplified equation subsets (IGP4 and SGP4) of AFGP4 are also given and the procedure by which they were obtained is outlined. The IGP4 and SGP4 theories were originally developed by Kenneth H. Cranford.