Fundamentals of Astrodynamics and Applications

By David A. Vallado

Tutorial Lectures at the

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Objectives

- Use an example problem to illustrate various astrodynamic techniques you'll need to know

- Introduce you to the various topics that the text covers in more detail

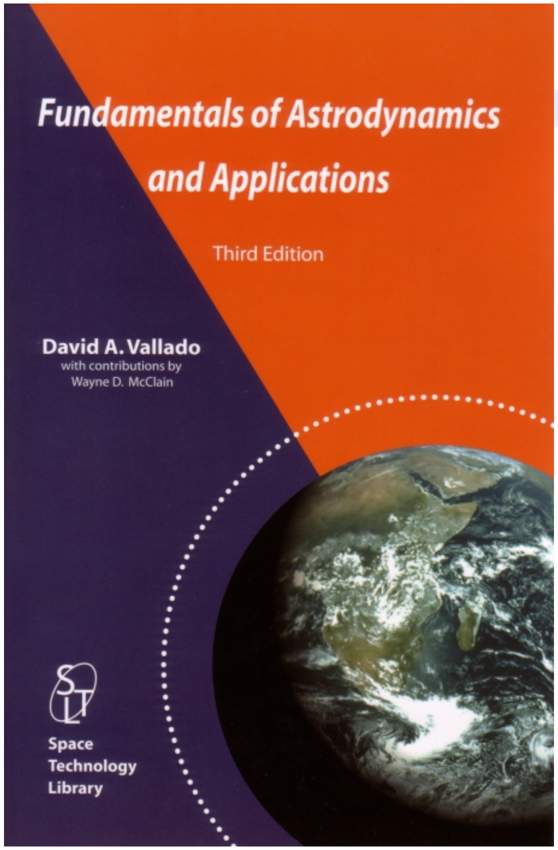
Problem Scenario

- Determine when you can see a satellite from a ground site

- What we'll need to understand
  - Time
  - Coordinate systems
  - Propagation
  - Orbit Determination
  - ... and some others ☺

What we'll cover

- Fundamental Concepts
  - Time and Coordinate Systems
- Newton
  - Equations of Motion
- Kepler
  - Equation
  - Problem
  - Satellite state
- Perturbations/Propagation
  - Special
  - General
- Orbit Determination and Estimation
- Applications



Fundamentals of Astrodynamics and Applications Third Edition

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By David A. Vallado

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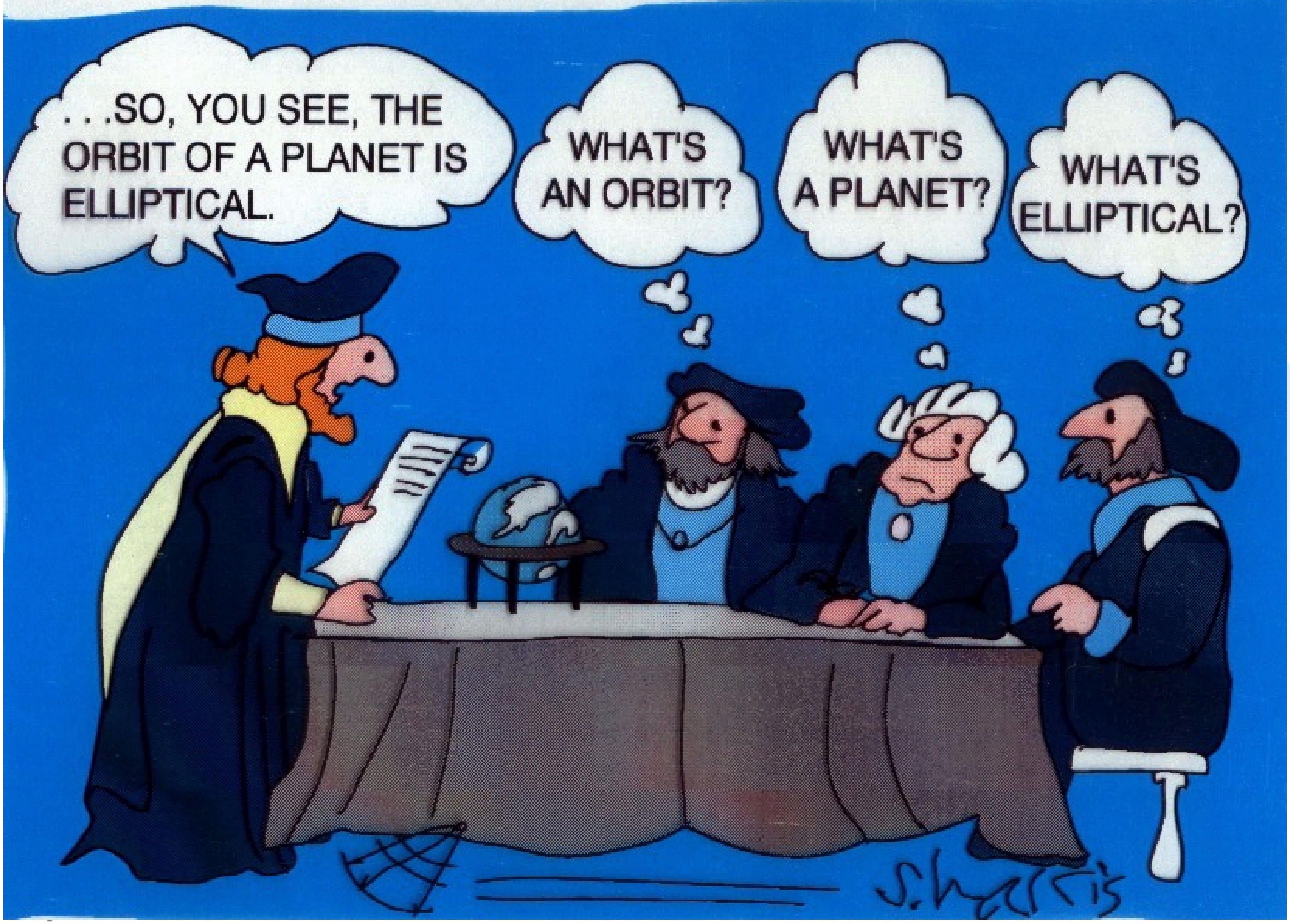
JOHANNES KEPLER'S UPHILL BATTLE



My Objective with the Book

- Cover
  - Fundamentals
  - Some advanced material
- Bridge the gap in between
- Details
  - Consistent notation

Chapter 3

- Fundamental Concepts
- Newton
- Kepler
- Perturbations
- Orbit Determination
- Applications

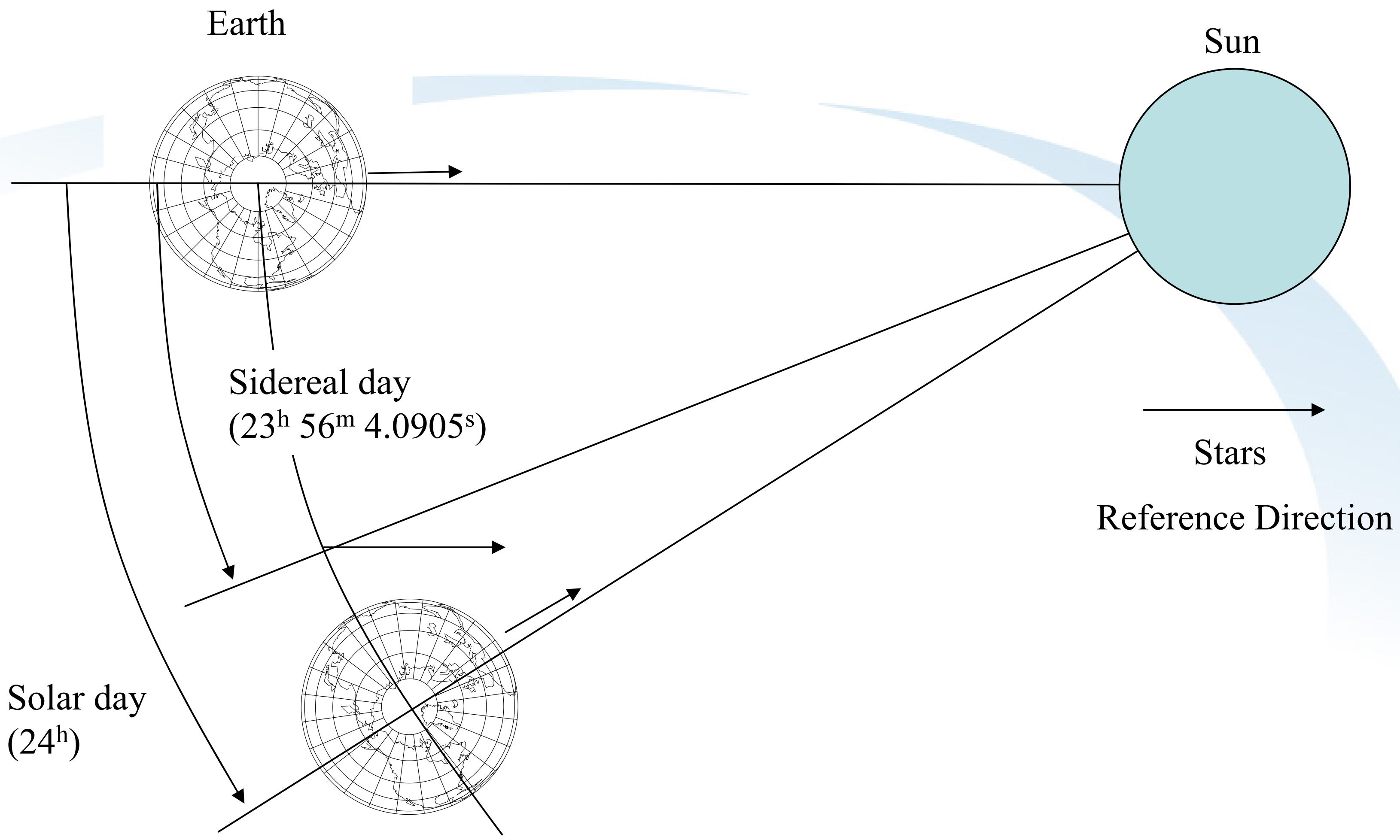
Time and Coordinate Systems

- Essential, but not terribly exciting

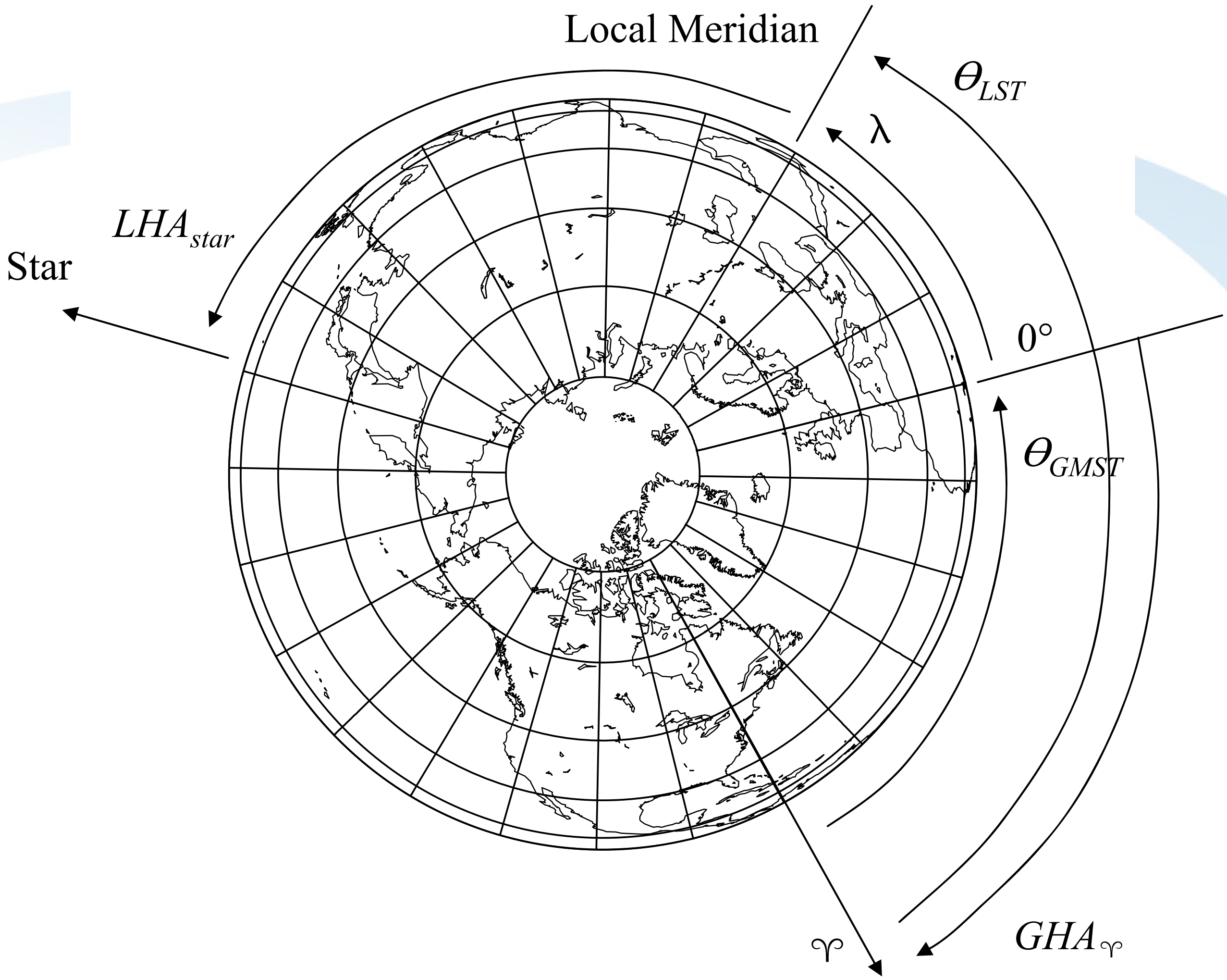
What Time is it?

- 14:28
  - Ok – That specifies that it's afternoon
  - But what time zone?
    - Mountain Time is 6/7 hours before UTC (Greenwich, Zulu)
      - Need to specify
        - » Daylight Savings
        - » Standard Time
  - Is that all? … No!
    - TAI, TT (TDT), TDB, TCB, TCG, GPS, …

Solar and Sidereal Time



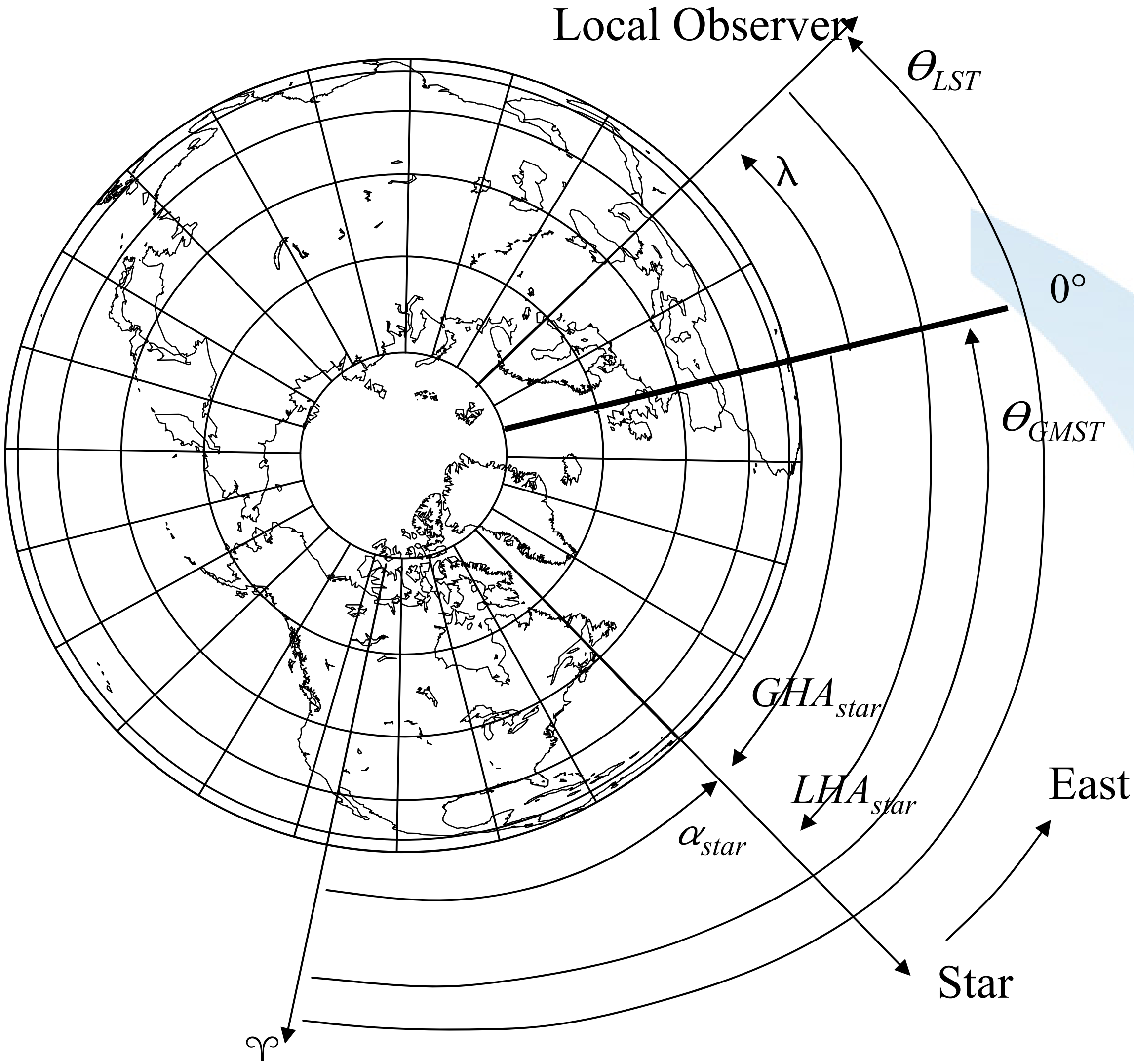
Greenwich and Local Times



Hour Angles vs Time?

- 24 hrs = 360 degrees
  - Sidereal time assumed

Hour Angles

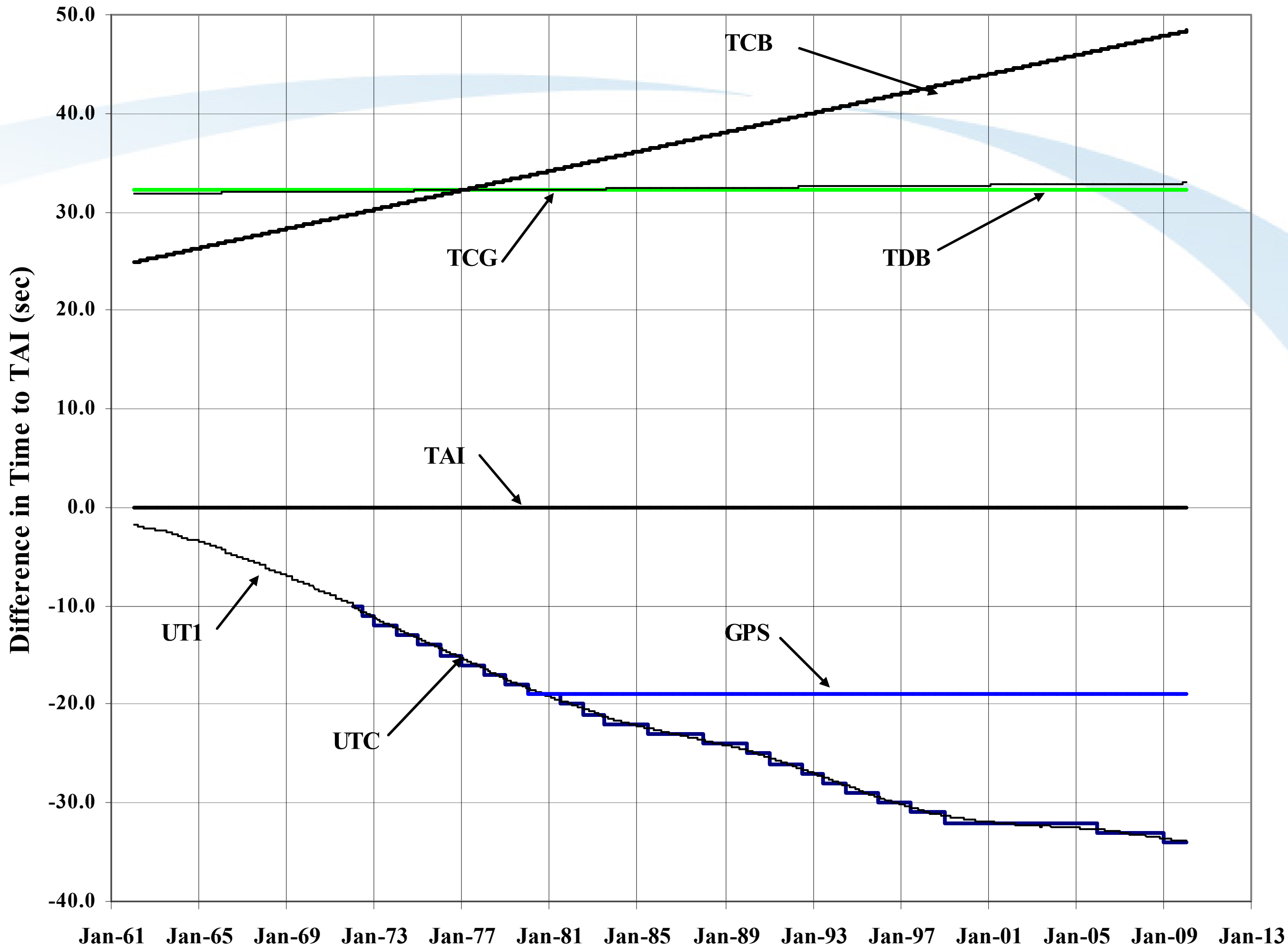


Local Observer
θ_{LST}
λ
0°
θ_{GMST}
GHA_{star}
LHA_{star}
α_{star}
East
Star
♈

What Time is it? (continued)

- Additional times
  - UT1 (Universal Time, sidereal time)
    - Solution from observations
    - Shows slowly decreasing Earth rotation rate
  - UTC is Coordinated Universal Time (solar time)
    - "Clock time"
    - Maintained within 0.9 s of UT1
      - Leap Seconds
    - $UTC = UT1 + \Delta UT1$
      - ΔUT1
        - » EOP Parameter that accounts for actual Earth rotation
        - » Calculated by USNO/ IERS

Time Scales



Difference in Time to TAI (sec)
50.0
40.0
30.0
20.0
10.0
0.0
-10.0
-20.0
-30.0
-40.0
TCB
TCG
TDB
TAI
UT1
GPS
UTC
Jan-61
Jan-65
Jan-69
Jan-73
Jan-77
Jan-81
Jan-85
Jan-89
Jan-93
Jan-97
Jan-01
Jan-05
Jan-09
Jan-13

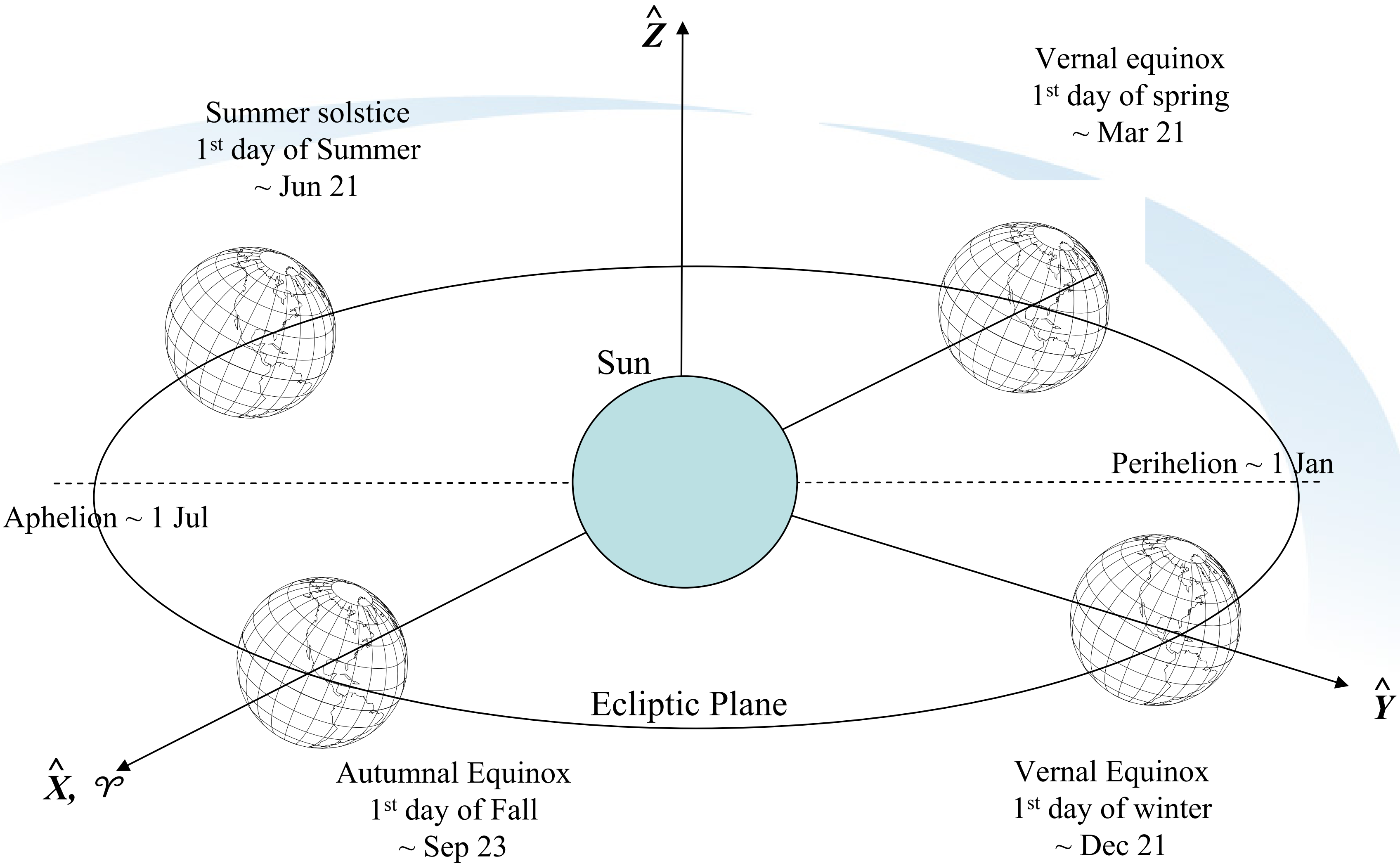
Summary for time

- Time
  - Can be off by up to a second if no ΔUT1
  - TT can be off a minute
    - Used for many calculations
  - Impact
    - Seems small but …
      - Consider satellite traveling at 7 km/s
  - Many conversions necessary
    - Satellite moves wrt sidereal time
    - Clocks record Solar time

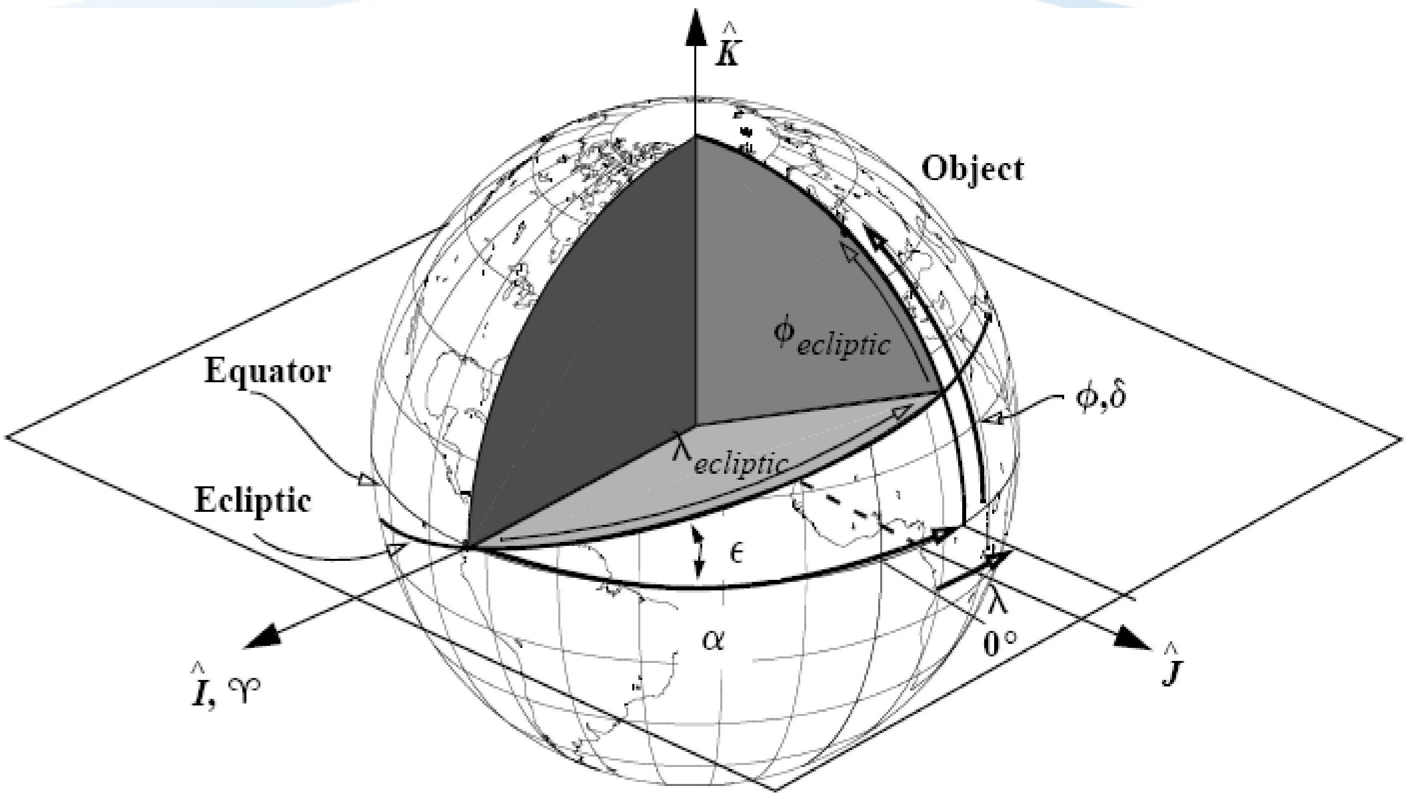
Coordinate Systems

- Sun based
  - Heliocentric
  - Barycentric
- Earth Based
  - Geocentric (Inertial and fixed)
  - Topocentric (fixed)
- Satellite Orbit Based
  - Perficoal
  - Radial vs Normal
  - Equinoctial
- Satellite Based
  - Attitude

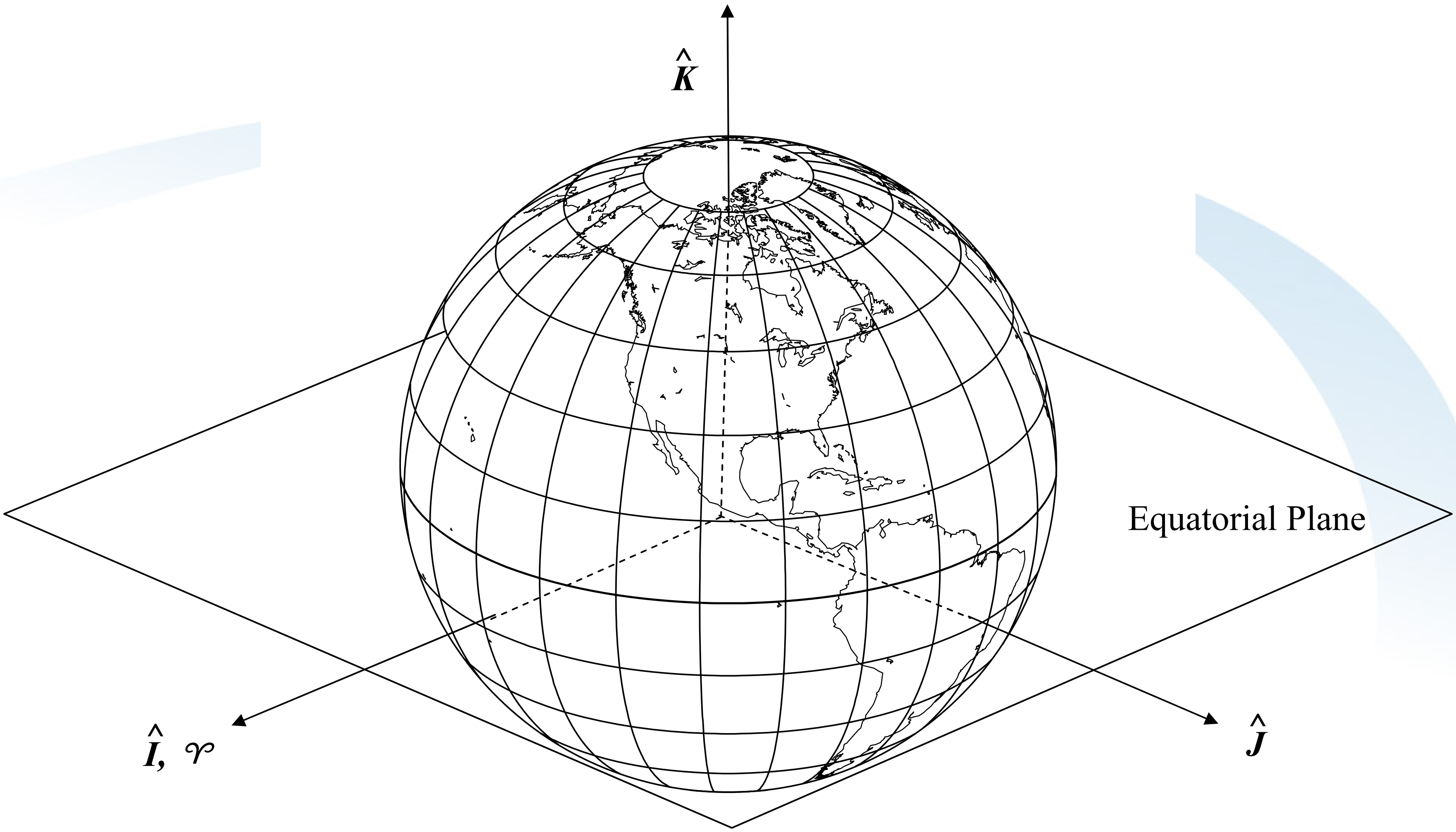
Heliocentric Coordinate System



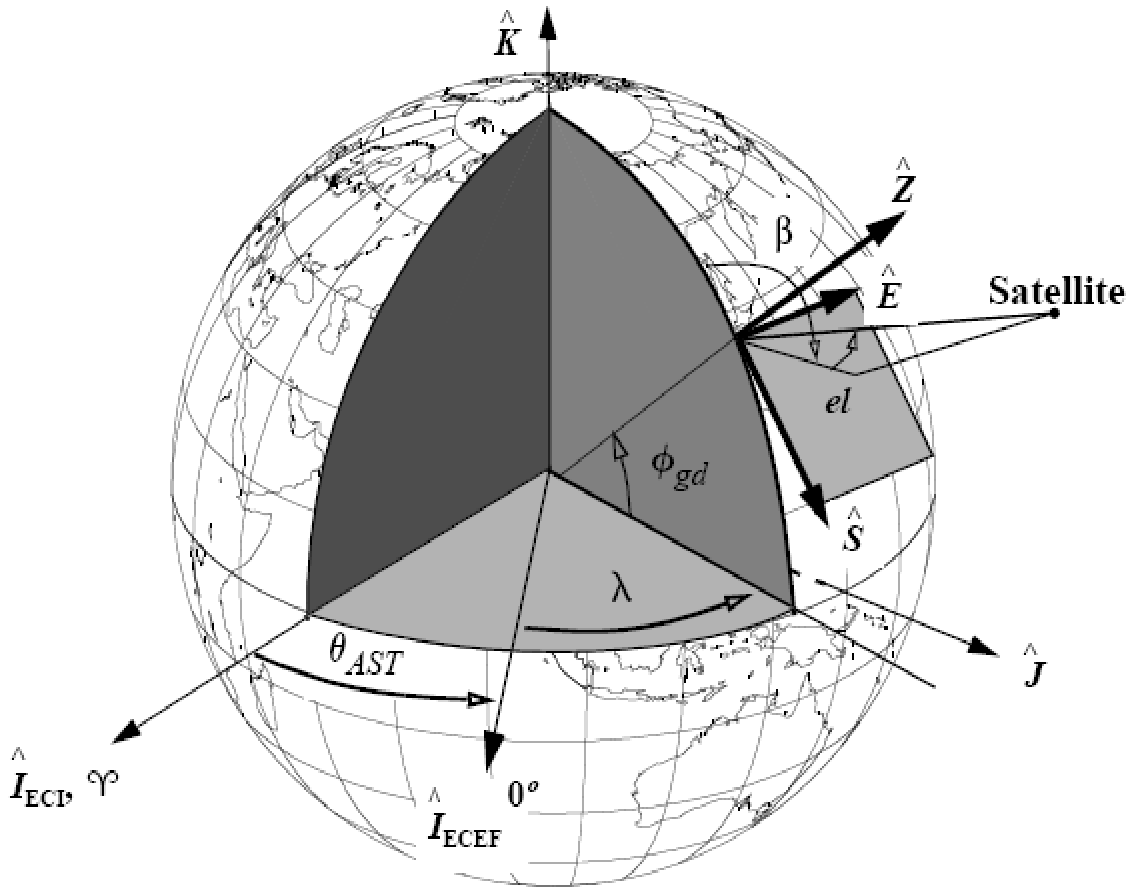
Geocentric and Ecliptic Coordinates



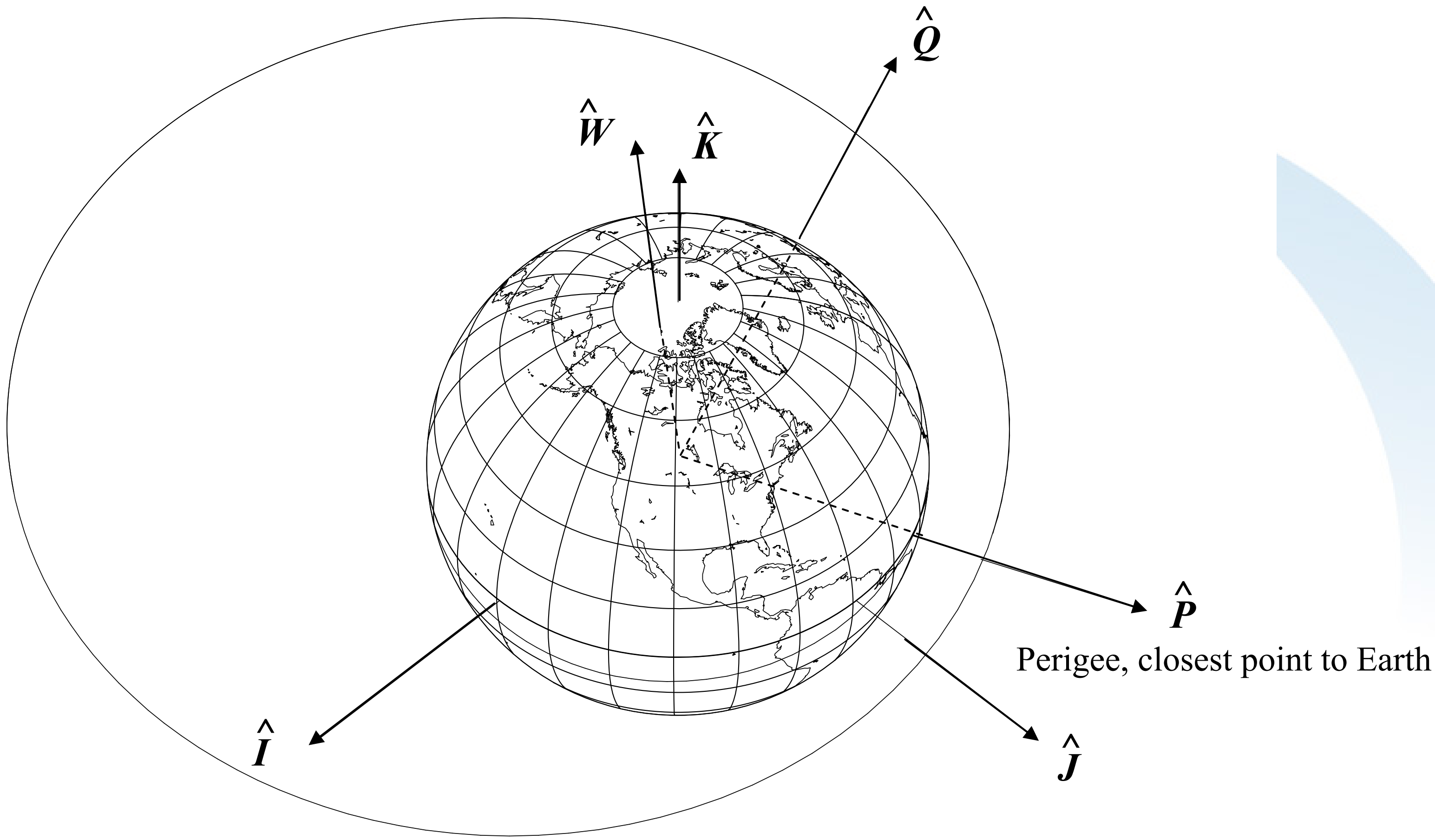
Geocentric Coordinate System



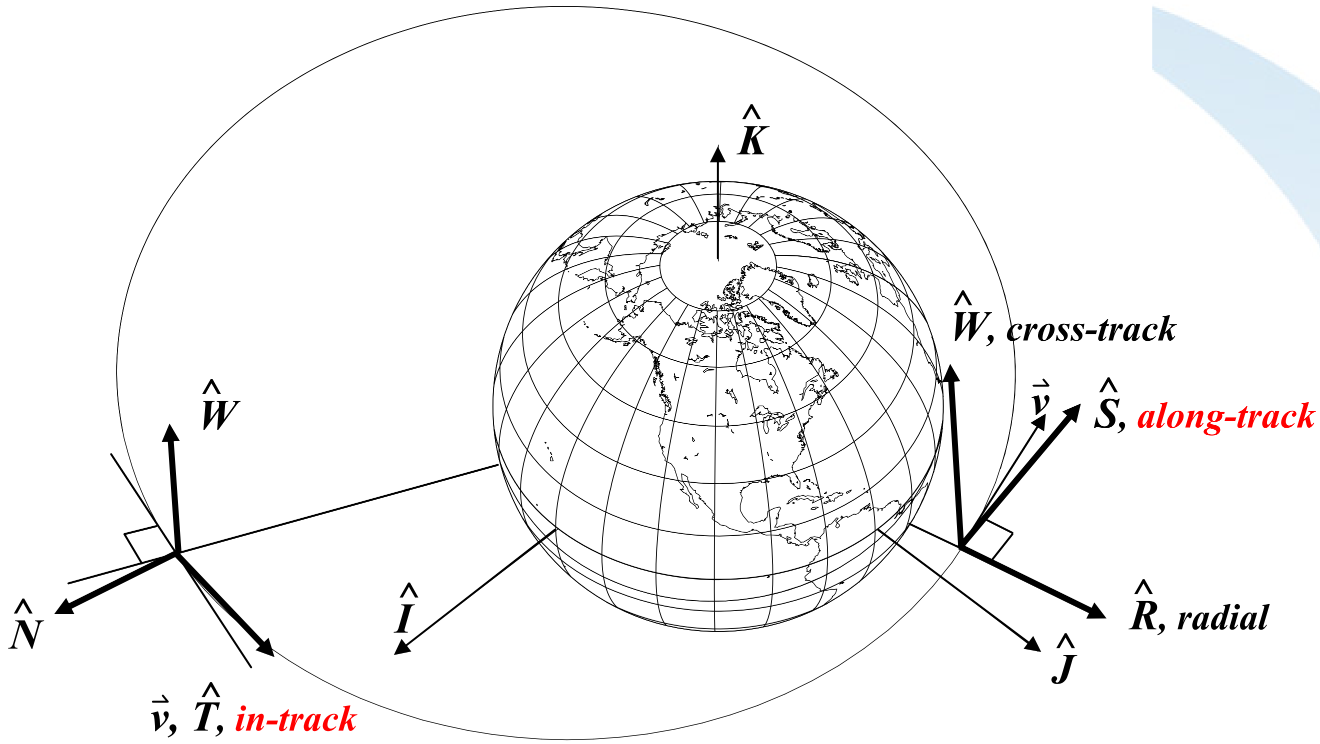
Local Coordinate System



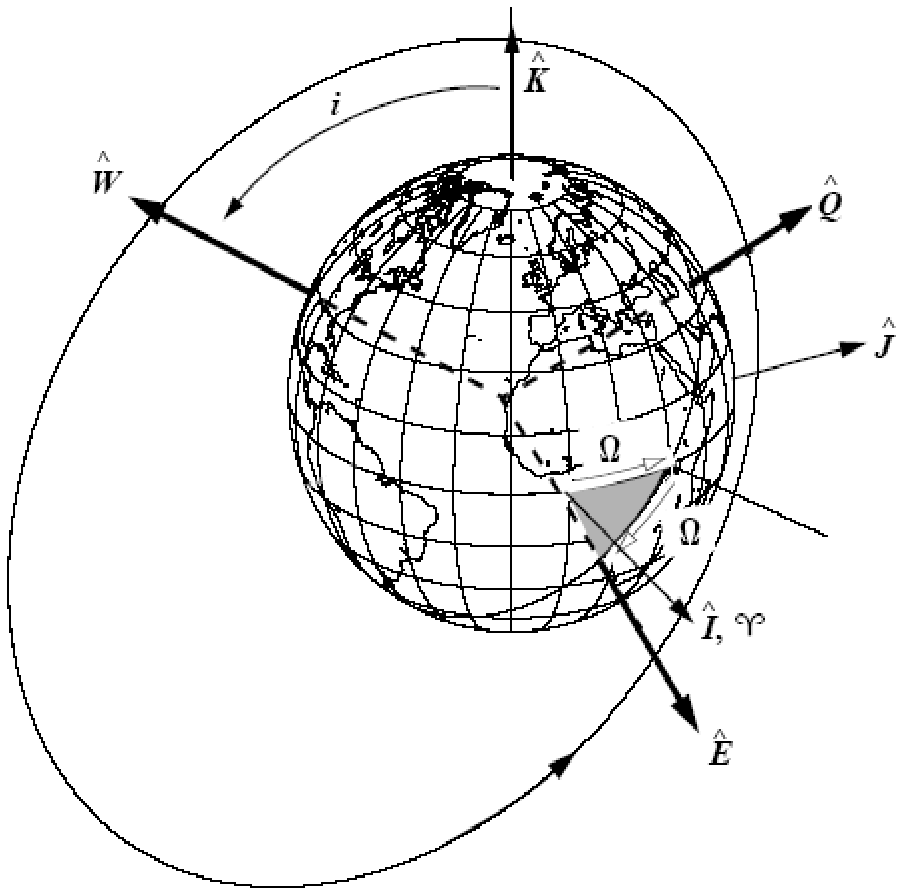
Orbit Based Systems - Perifocal



Orbit Based Systems – Normal and Radial



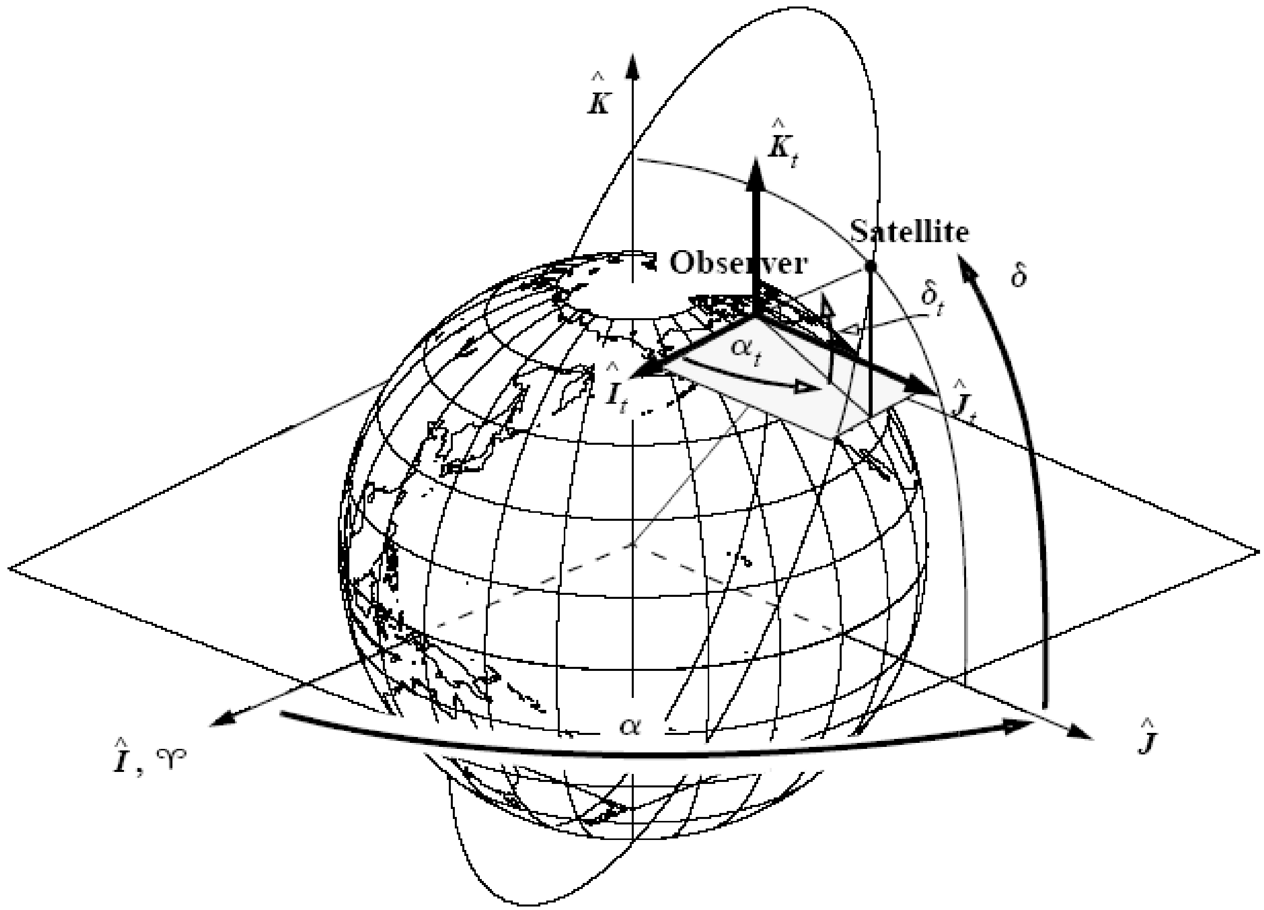
Orbit Based Systems - Equinoctial



Angular Measurements

- Latitude and longitude
  – Familiar
- Right Ascension-Declination
  – Optical measurements

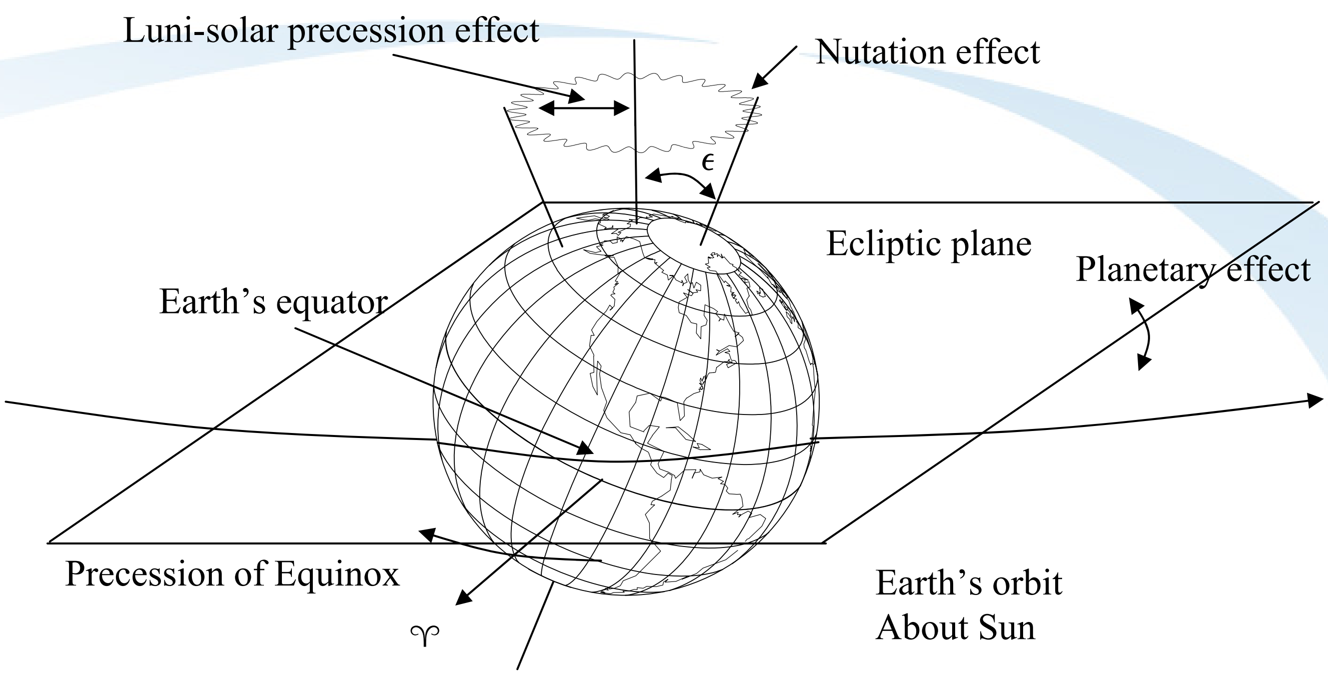
Right Ascension - Declination



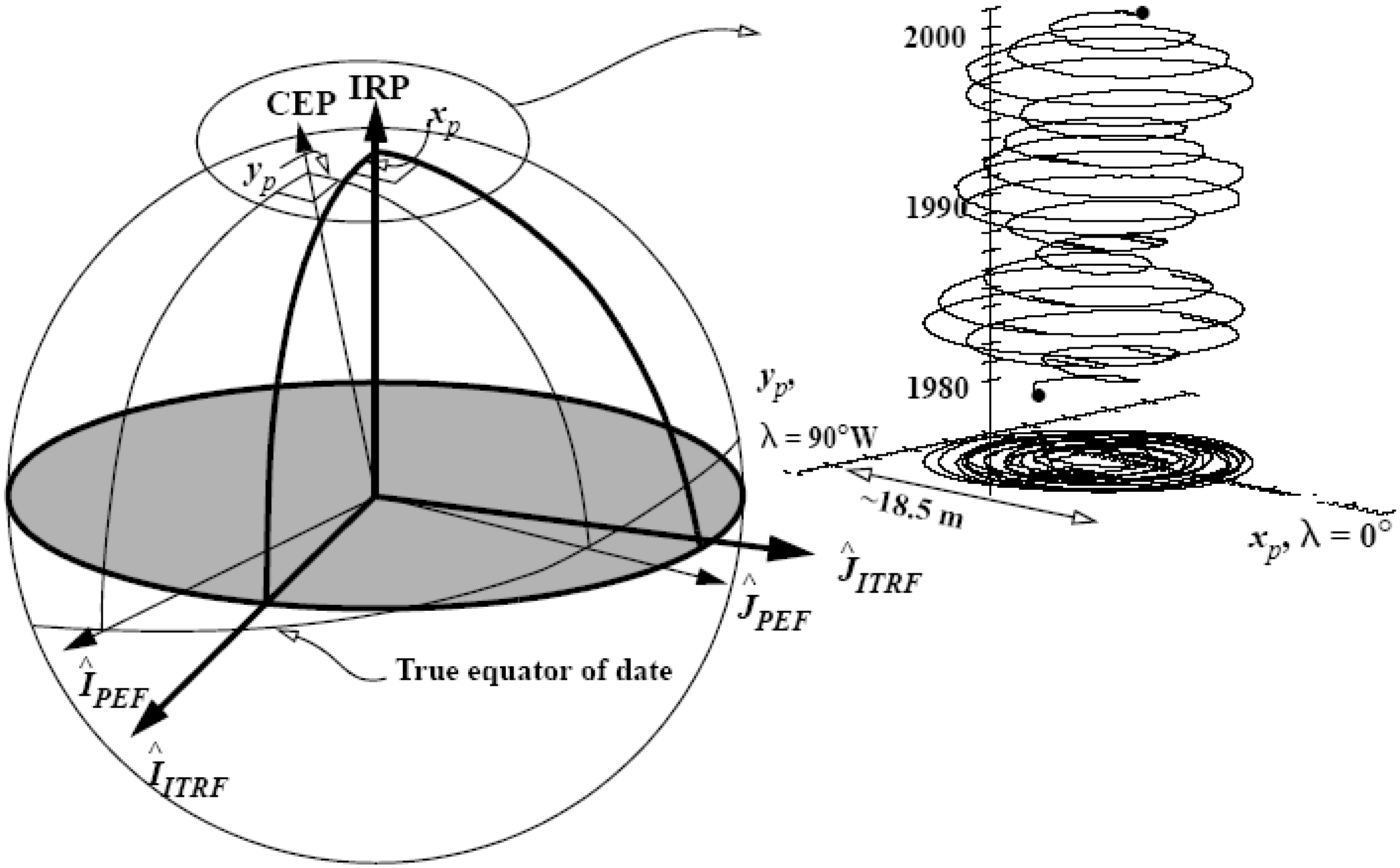
Motion of the Coordinate System

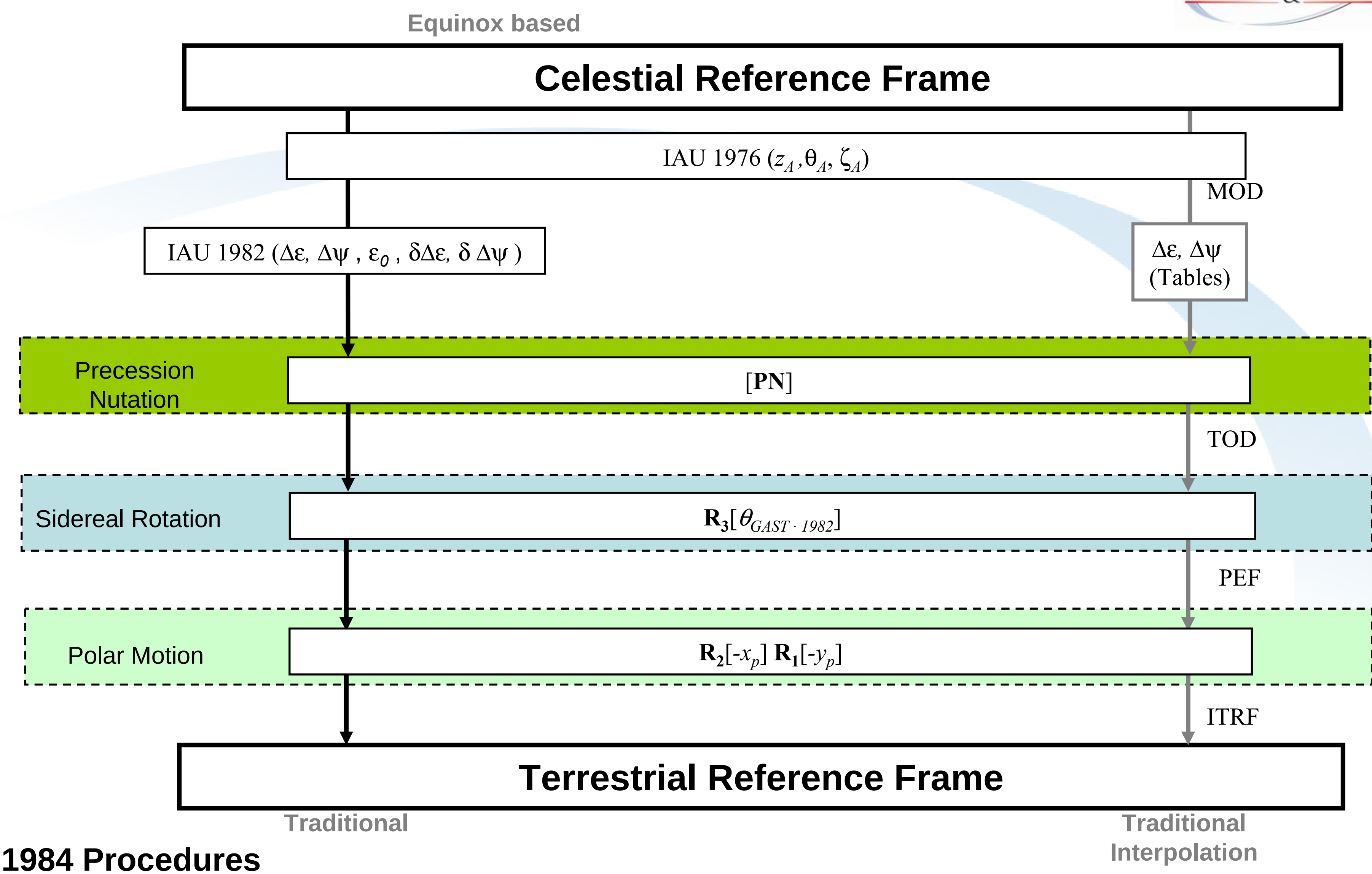
- Earth's orbit is not exactly stable
  - Precession
    - Long period movement (~26000 years)
  - Nutation
    - Short period movement (~18.6 years)
- Fixed vs Inertial
  - Sidereal Time
- Polar Motion
  - Axis of rotation moves slightly over time

Precession and Nutation

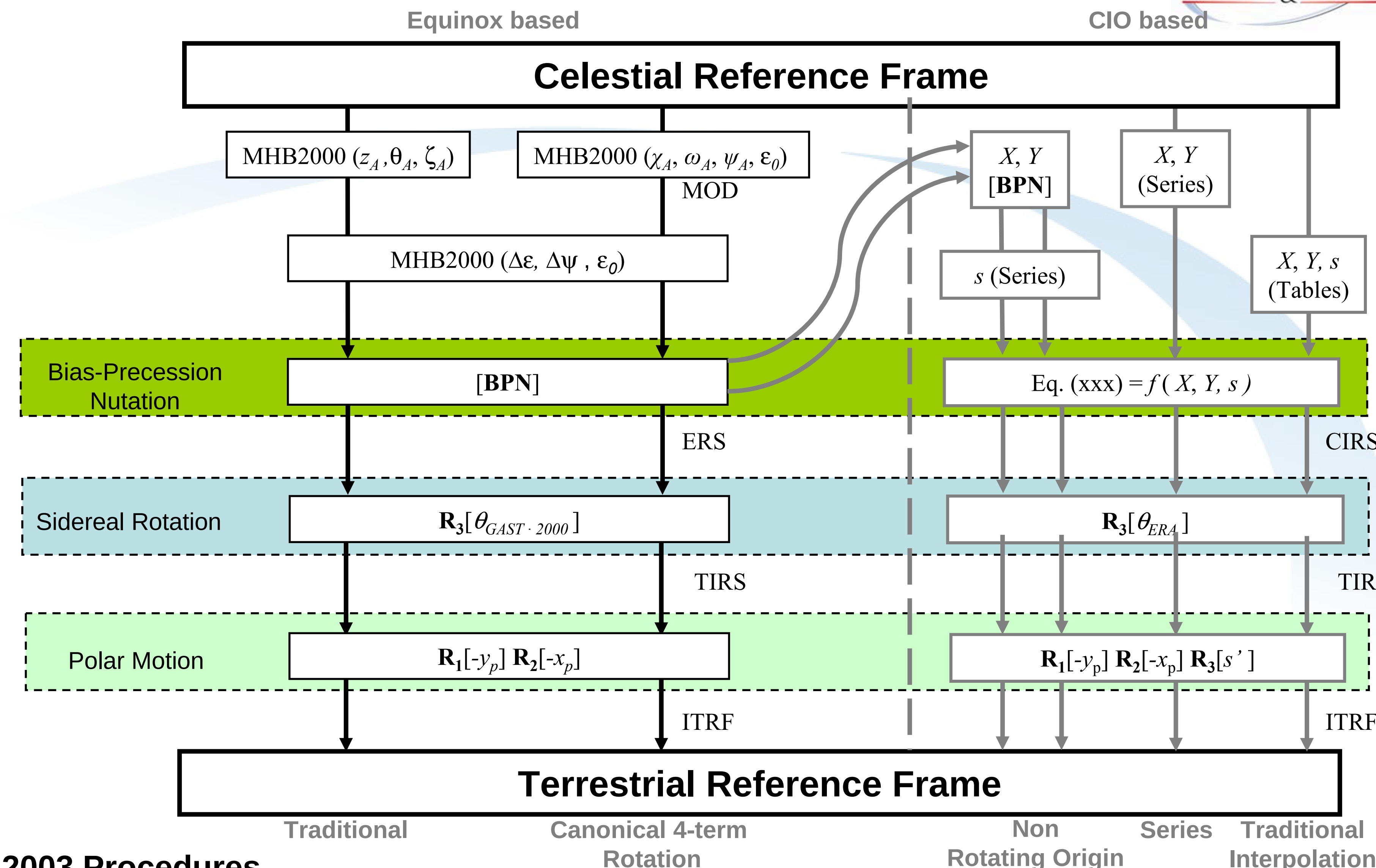


Polar Motion

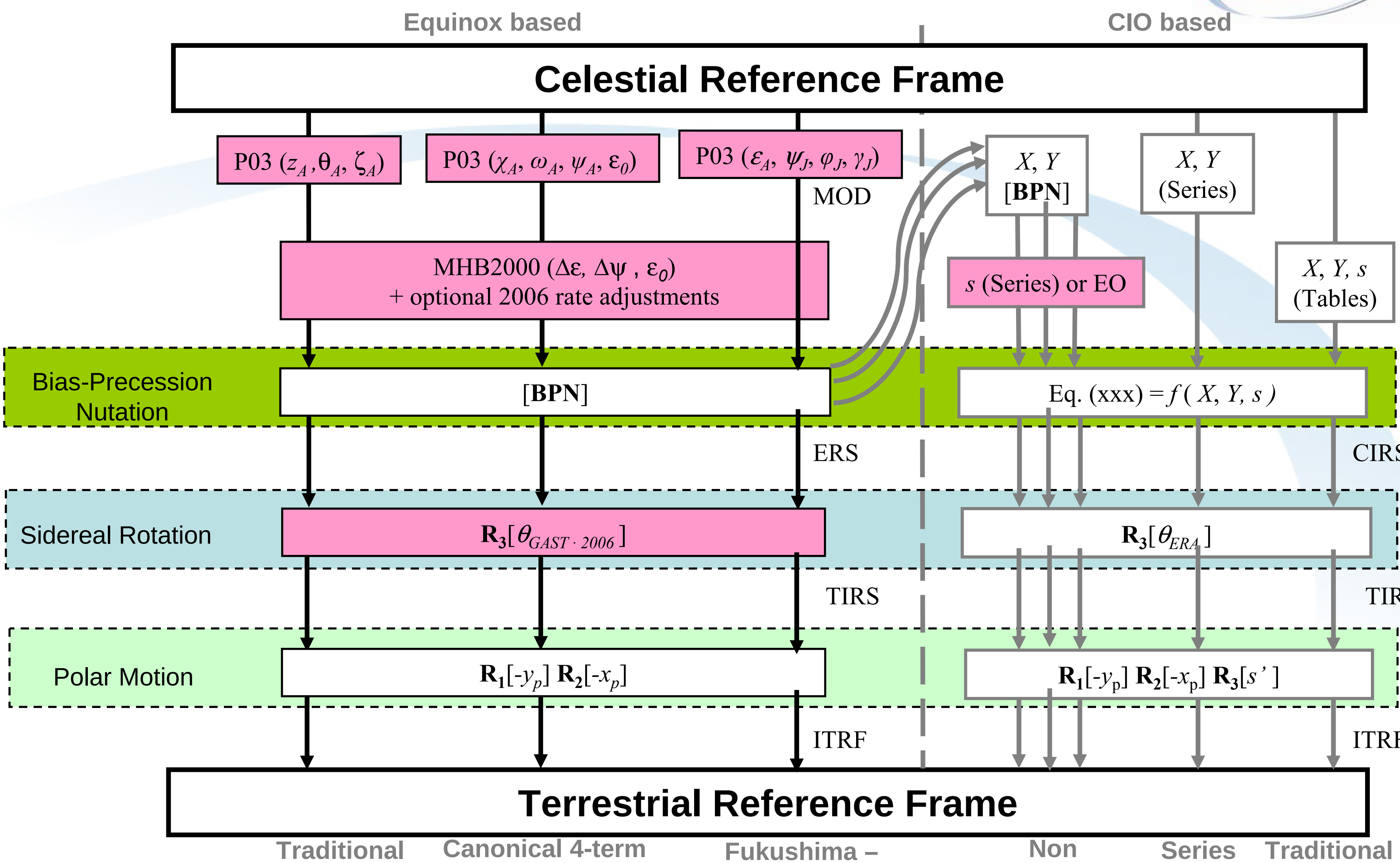




1984 Procedures



2003 Procedures

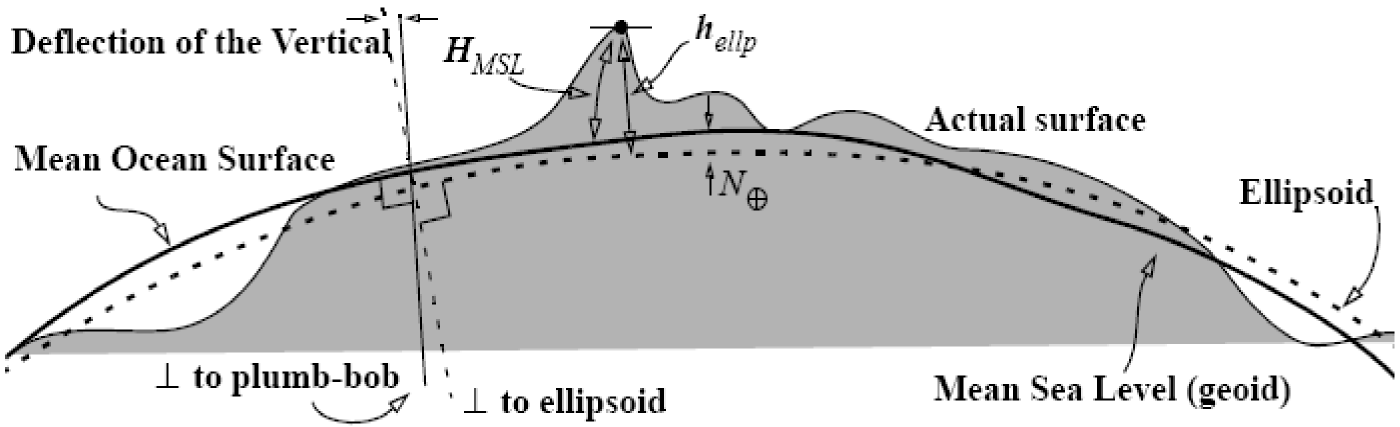


2006 Procedures

Earth’s shape

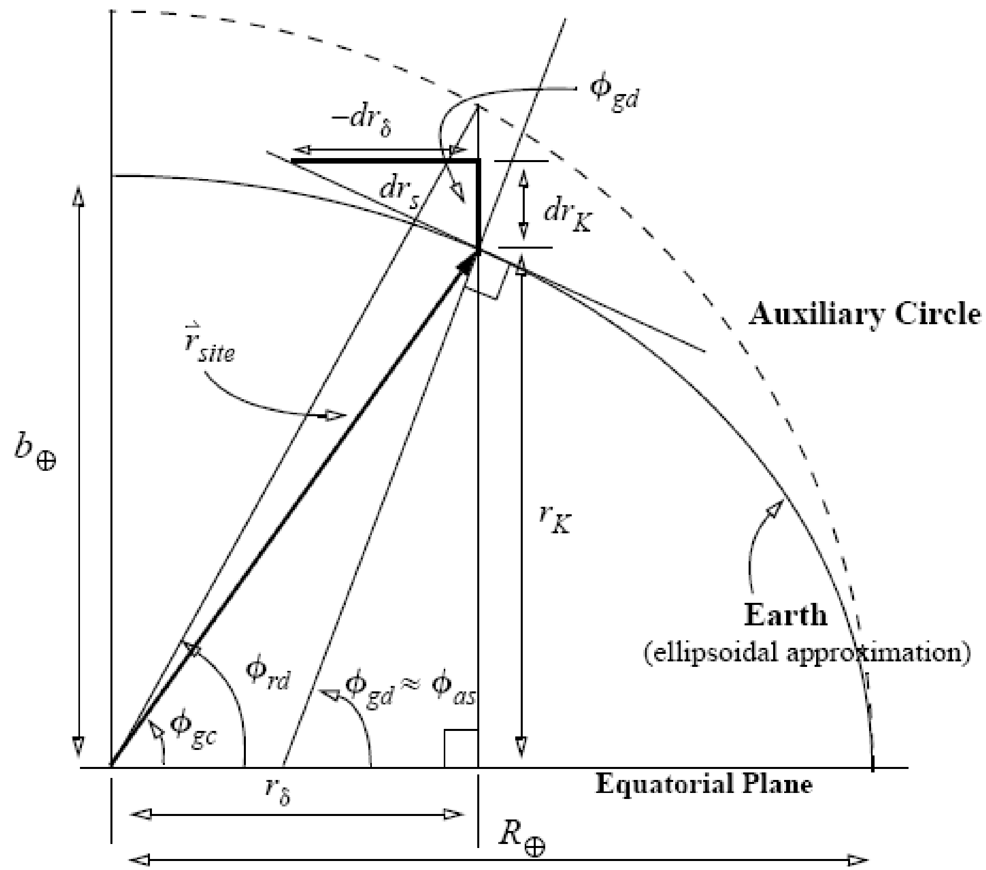
- Oblate Spheroid
  - An ellipsoidal approximation
- Other terms
  - Geoids
    - Gravity acts equally at all points on this surface
      - Plumb-bobs will hang perpendicular
  - Geopotential
    - Mathematical representation of the precise gravitational effect

Earth Surface



Earth Ellipsoid

- Convert geocentric (ϕ_{gc}) and geodetic (ϕ_{gd}) latitude

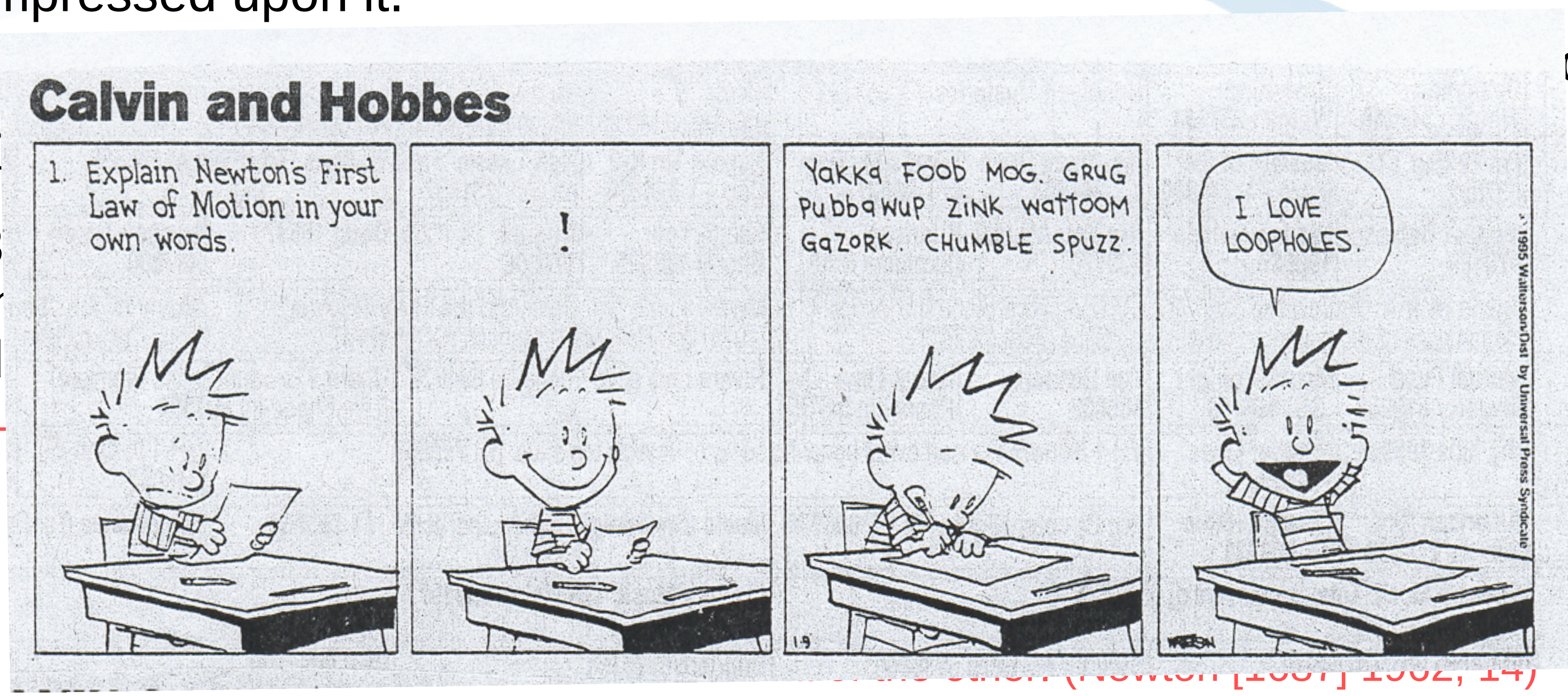


Chapter 1

- Fundamental Concepts
- Newton
- Kepler
- Perturbations
- Orbit Determination
- Applications

Newton’s Laws

- 1. Every body continues in its state of rest, or of uniform motion in a right [straight] line, unless it is compelled to change that state by forces impressed upon it.
- 2 … d a … ir …
- 3 … m … d …
  - … f … of the other (Newton [1687] 1962, 14)



Gravitational Law

- Forms the basis of Two-body dynamics
  - G is constant of gravitation = 6.673×10^{-20} km^3/kgs^2

$$\vec{f}_{gravity} = -\frac{G m_{\oplus} m_{sat}}{r^2} \frac{\vec{r}}{|\vec{r}|}$$

Two-body Equation of Motion

- Simple form resulting from

$$\ddot{\vec{r}} = -\frac{G(m_{\oplus} + m_{sat})}{r^2}\frac{\vec{r}}{|\vec{r}|}$$

- Fundamental Concepts
- Newton
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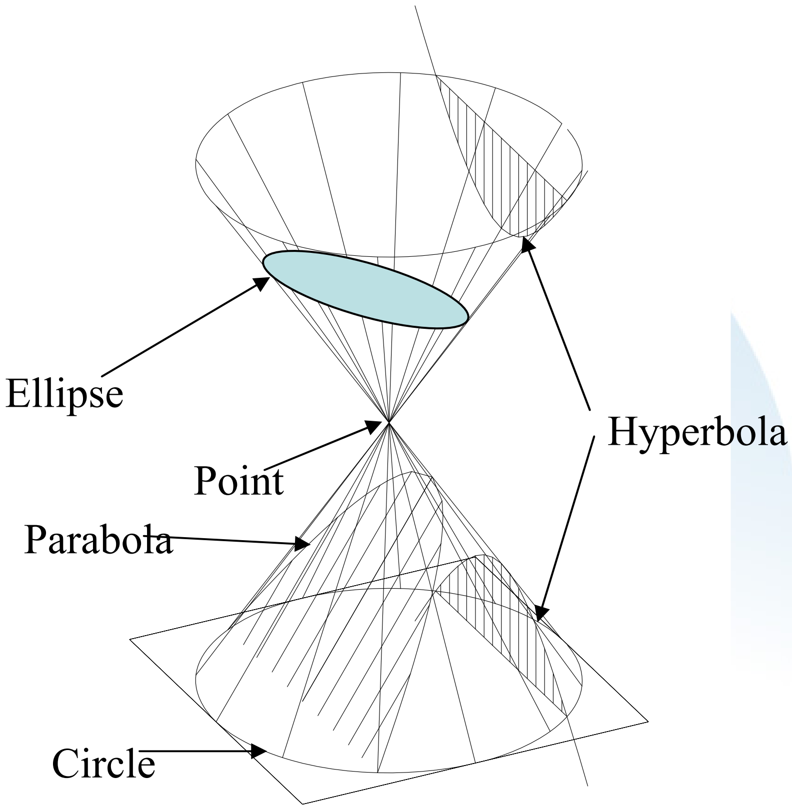
Chapter 2

Kepler's Laws

- 1. The orbit of each planet is an ellipse with the Sun at one focus.
- 2. The line joining the planet to the Sun sweeps out equal areas in equal times.
- 3. The square of the period of a planet is proportional to the cube of its mean distance to the Sun.

Conic Sections

- All orbits follow
  - Circle
  - Ellipse
  - Parabola
  - Hyperbola
  - Rectilinear

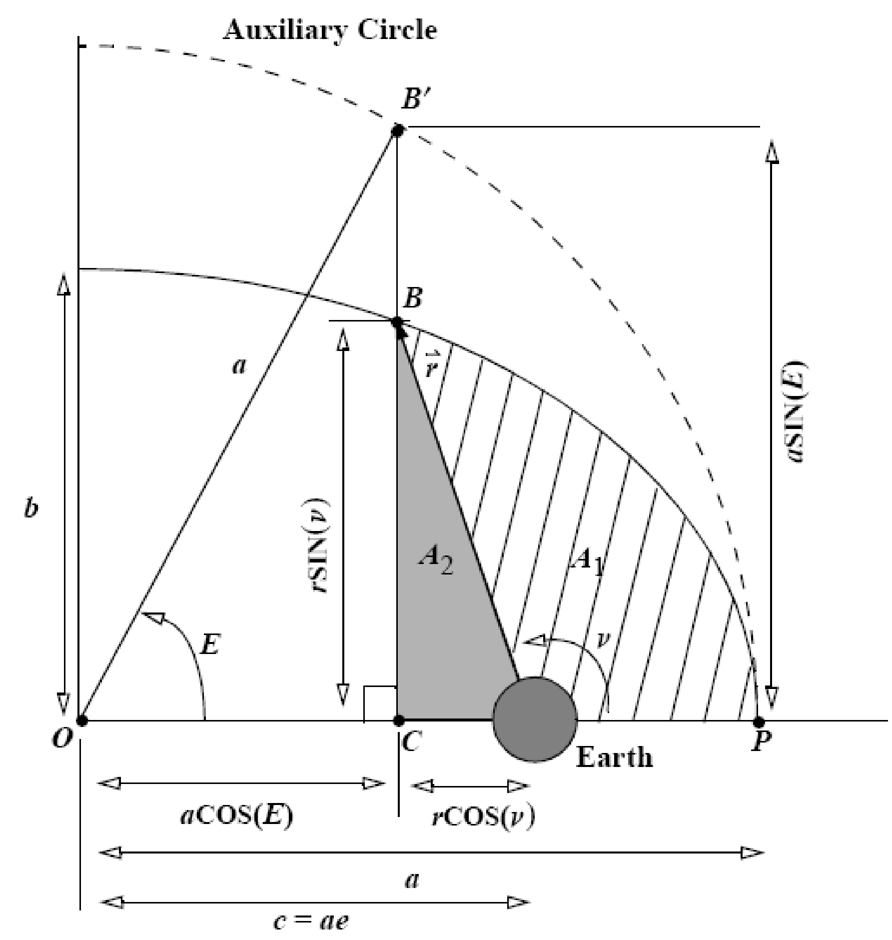


Nomenclature

- Kepler's Equation and Kepler's Problem
  - Very different!
  - Kepler's equation
    - Found during Kepler's analysis of the orbit of Mars
  - Kepler's problem
    - Generically used for propagating a satellite forward
      - Usually two-body dynamics

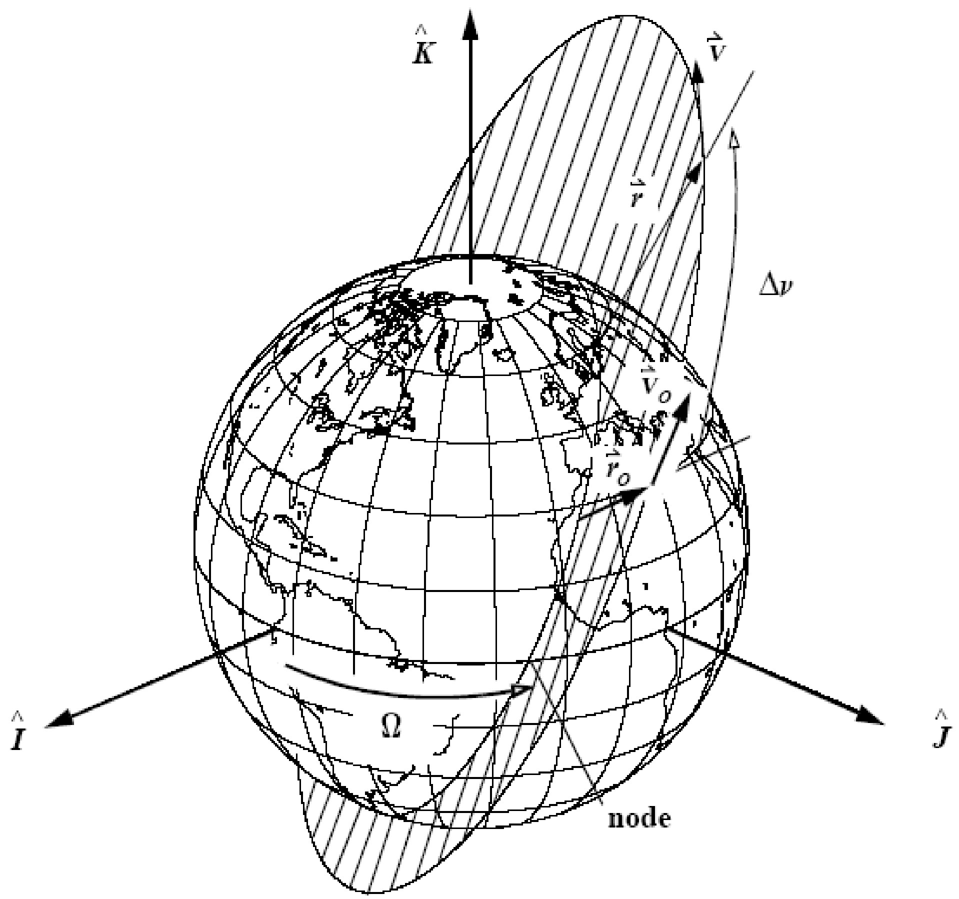
Kepler's Equation

- Find Eccentric anomaly (E)
  - $E = 0^o$ at $\nu = 0^o, 180^o$



Kepler's Problem

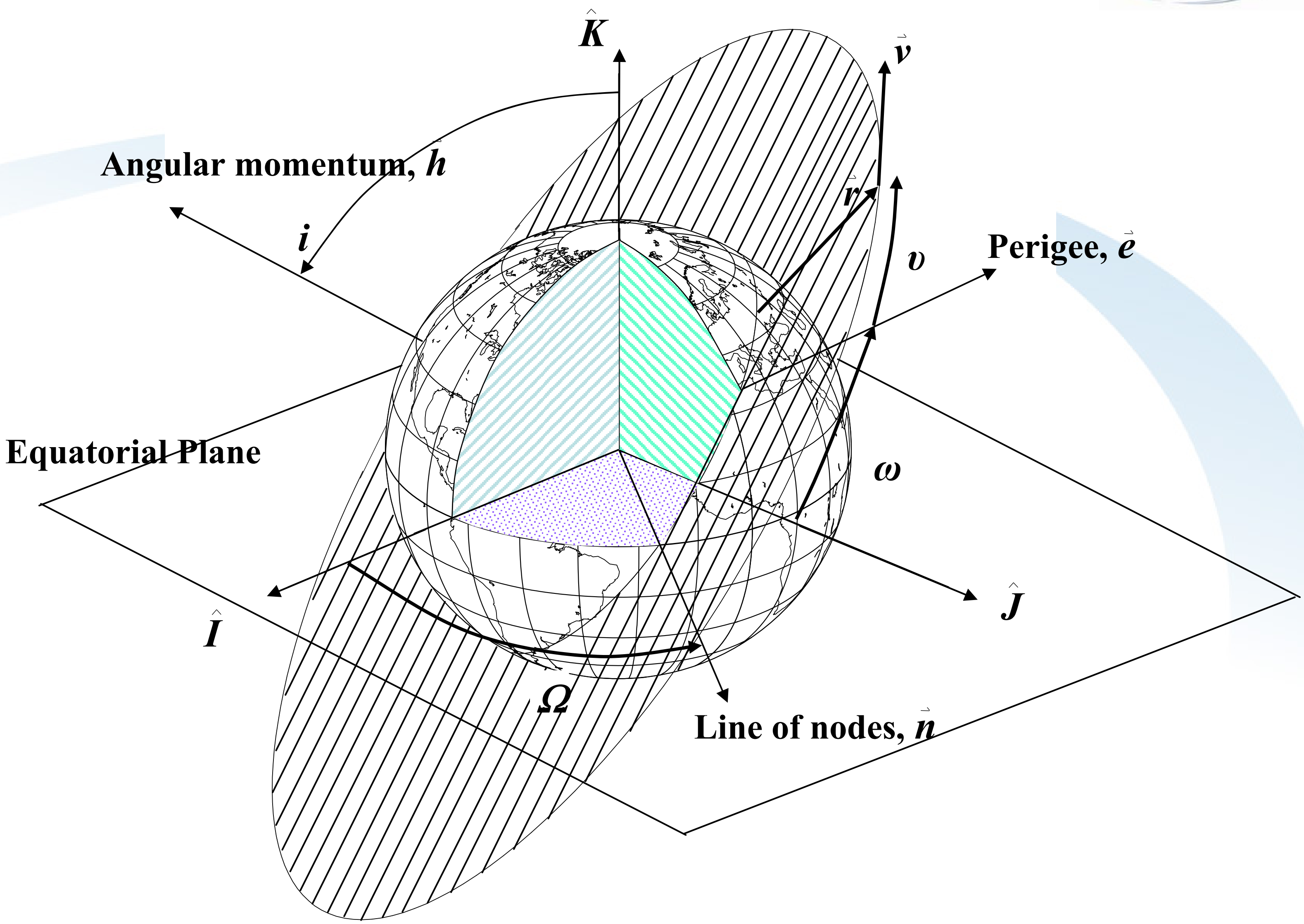
- Find future position and velocity
  - Given starting state
  - Called propagation



Satellite State Representations

- Convey location of a satellite in space and time
- Types
  - Numerical
    - Position and velocity vectors
  - Analytical (Elements)
    - Classical (Keplerian, Osculating, two-body) $(a, e, i, \Omega, \omega, \nu)$
    - Equinoctial $(a_f, a_g, L, n, \chi, \psi)$
    - Flight $(\lambda, \phi_{gc}, \phi_{fpa}, \beta, r, v)$
    - Spherical $(\alpha, \delta, \phi_{fpa}, \beta, r, v)$
    - Canonical
      - Delaunay
      - Poincare
    - Mean elements (theory dependant)
      - Two-line element sets
        - AFSPC, SGP4 derived, ‘mean’ elements
      - ASAP
      - LOP
      - Other
    - Other
  - Semianalytical
    - Theory dependant

Classical Orbital Elements



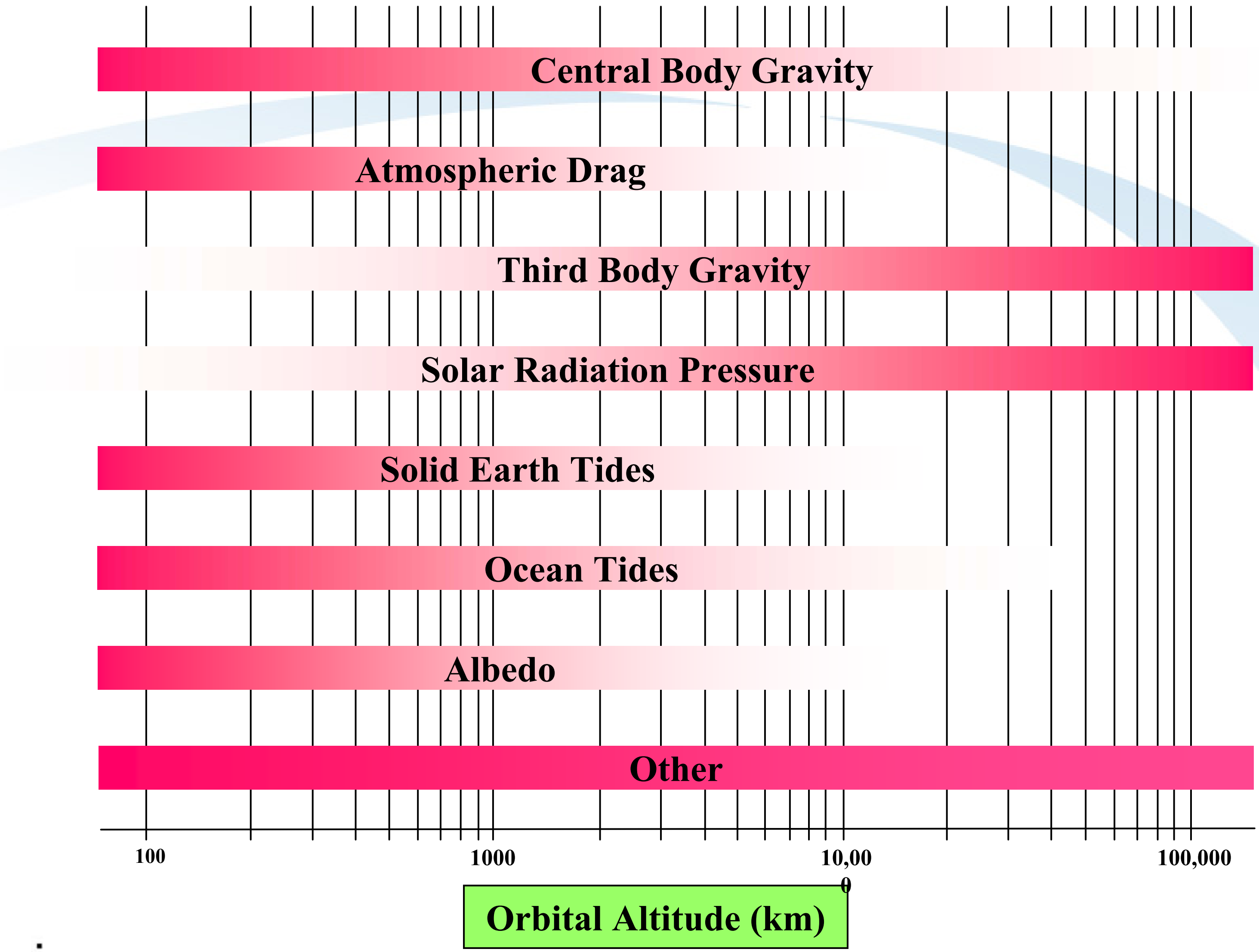
- Fundamental Concepts
- Newton
- Kepler
- Perturbations
- Orbit Determination
- Applications

Chapter 8/9

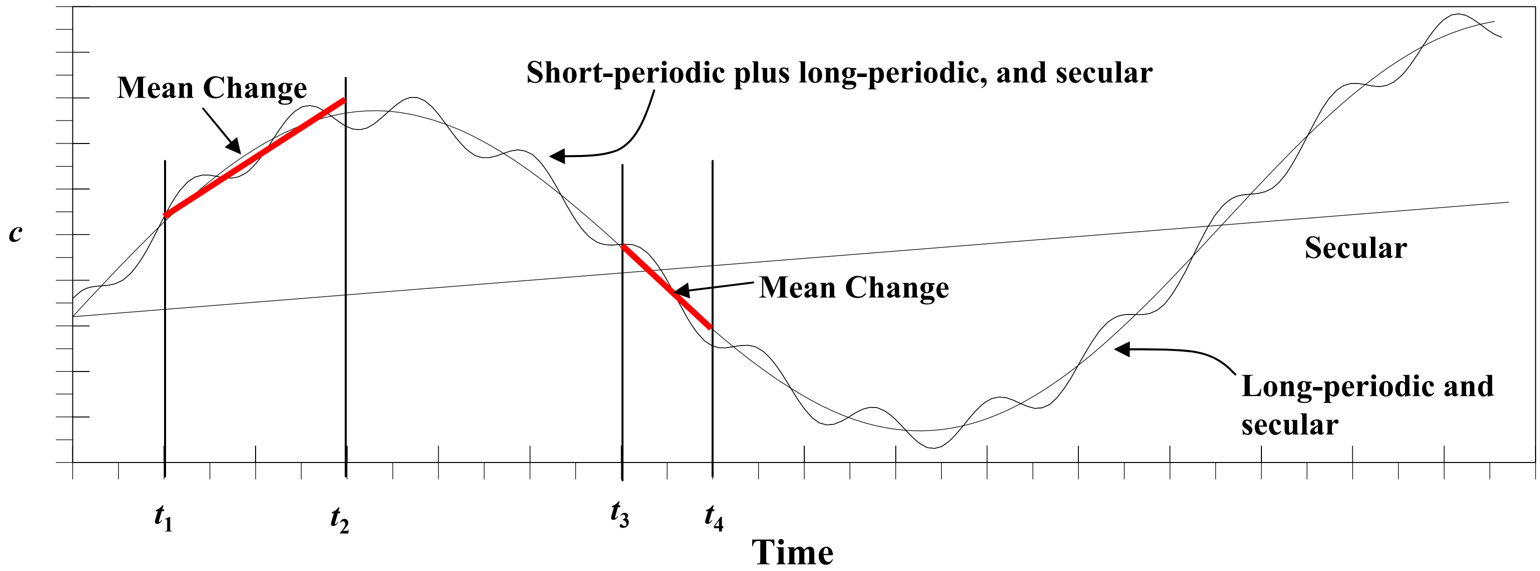
Introduction

- Several forces affect satellite orbits
  - Gravitational
  - Atmospheric Drag
  - Third Body
    - Sun, Moon, planets
  - Solar Radiation Pressure
  - Tides
    - Solid Earth, Ocean, pole, etc.
  - Albedo
  - Thrusting
  - Other

Applicability



Perturbations



Central Body Gravitational Forces

- Largest single contributor to the motion
  - It's why satellites stay in orbit!
- Conservative force
  - Total kinetic and potential energy remains the same

$$V = \frac{GM}{r}\left[1 + \sum_{n=2}^{\infty}\sum_{m=0}^{n}\left(\frac{a}{r}\right)^{n} P_{nm}(\sin\phi)(C_{nm}\cos m\lambda_{E} + S_{nm}\sin m\lambda_{E})\right]$$

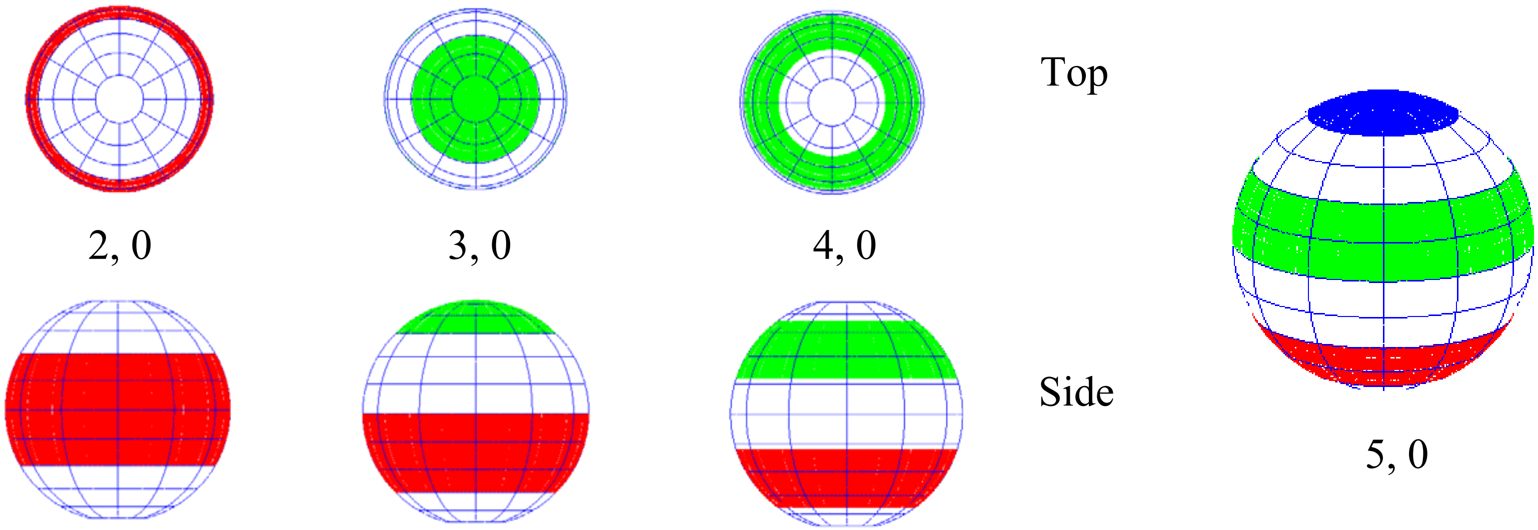
$$P_n(\sin\phi) = \frac{1}{2^n\, n!}\frac{d^m}{d^m(\sin\phi)}\left(\sin^2\phi - 1\right)^n$$

$$P_{nm}(\sin\phi) = (\cos\phi)^m \frac{d^m}{d^m(\sin\phi)} P_n(\sin\phi)$$

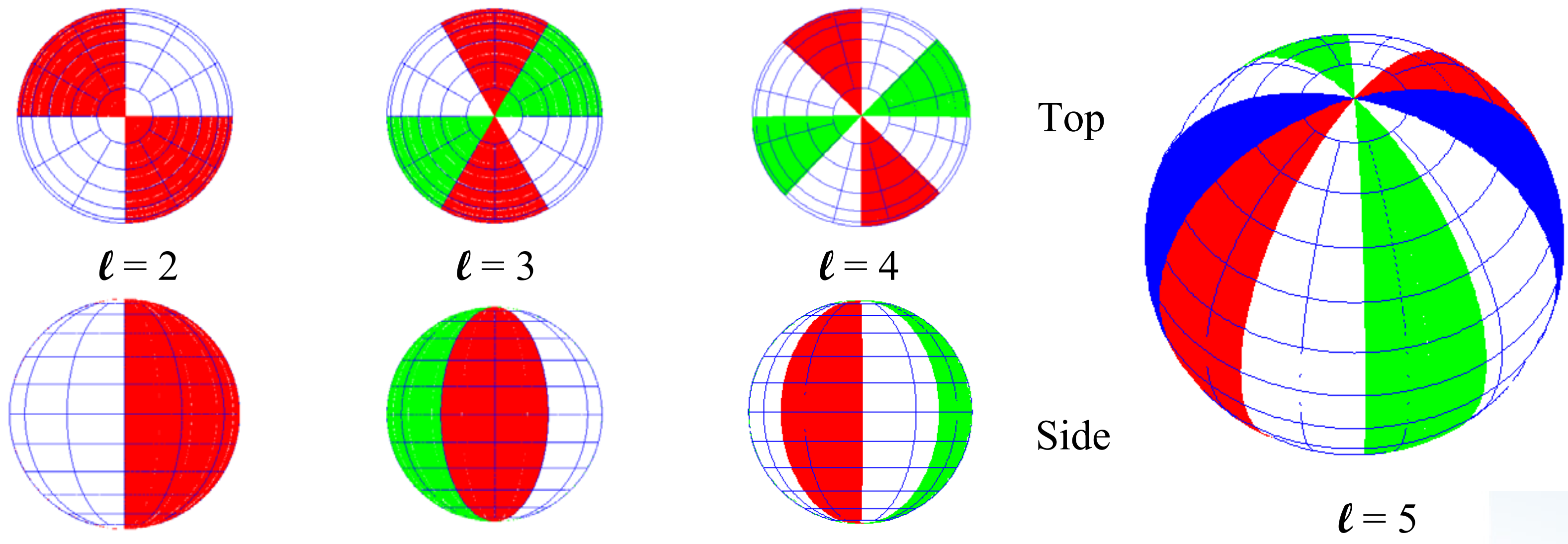
$$\bar{P}_{nm} = \left[\frac{(2n+1)k(n-m)!}{(n+m)!}\right]^{\frac{1}{2}} P_{nm}, \quad and \quad \begin{Bmatrix}\bar{C}_{nm}\\ \bar{S}_{nm}\end{Bmatrix} = \left[\frac{(n+m)!}{(2n+1)k(n-m)!}\right]^{\frac{1}{2}}\begin{Bmatrix}C_{nm}\\ S_{nm}\end{Bmatrix},$$

with $k = 1$ for $m = 0$, and $k = 2$ for $m \neq 0$.

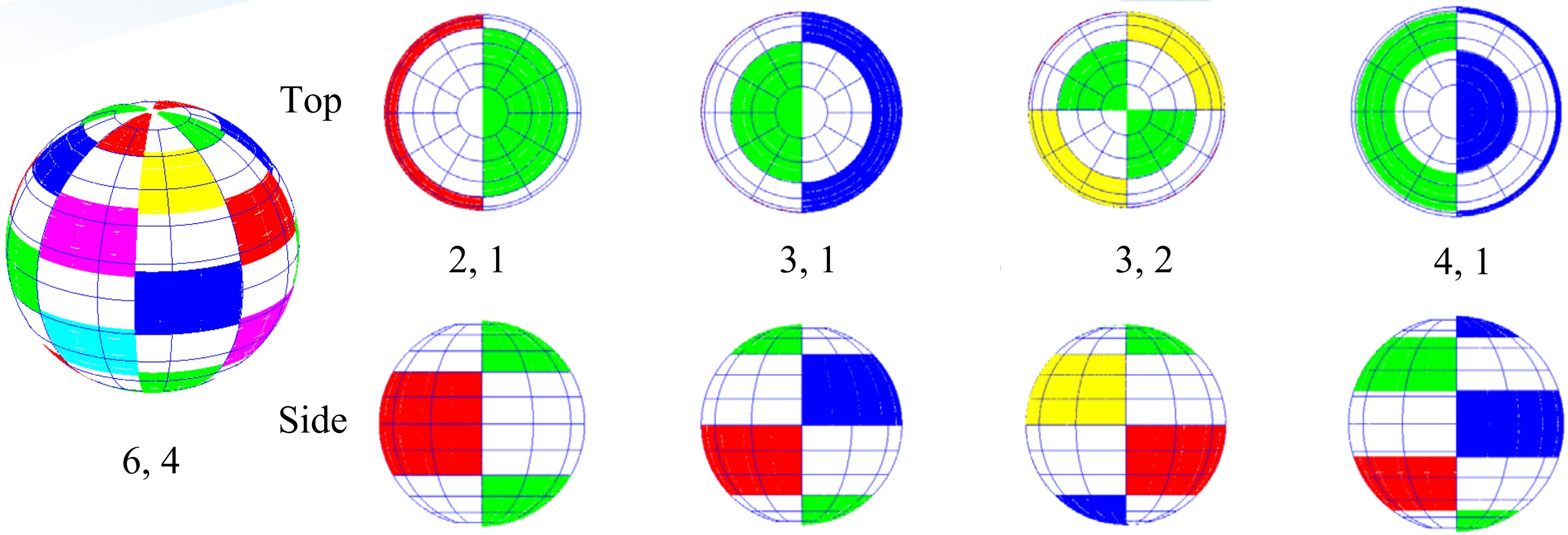
Zonal Harmonics



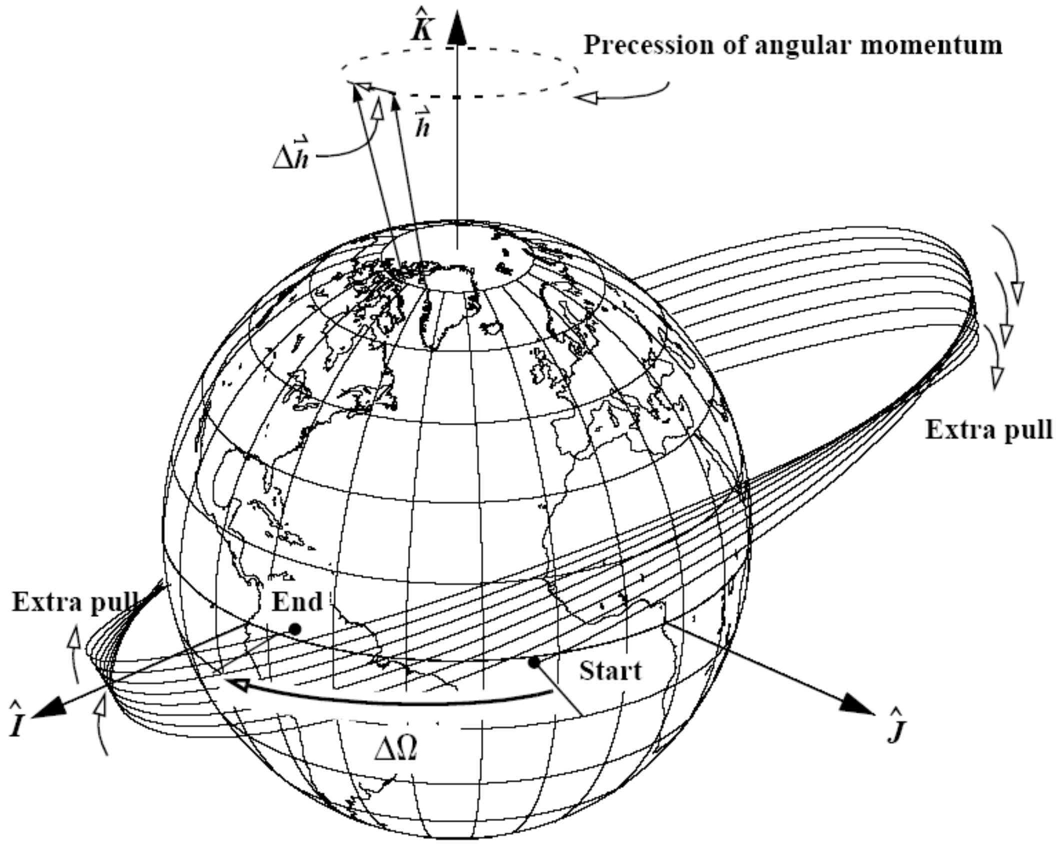
Sectoral Harmonics



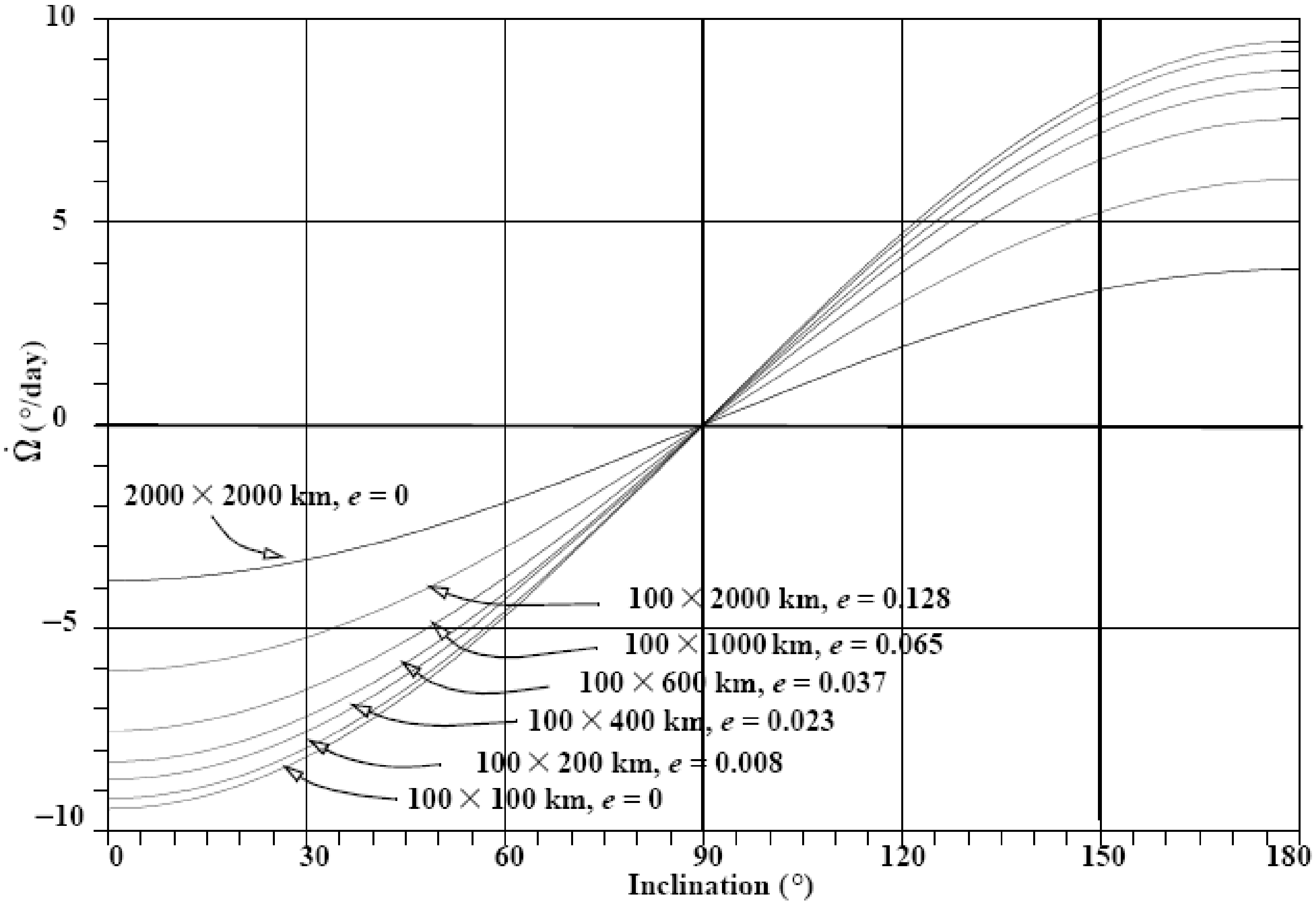
Tesseral Harmonics



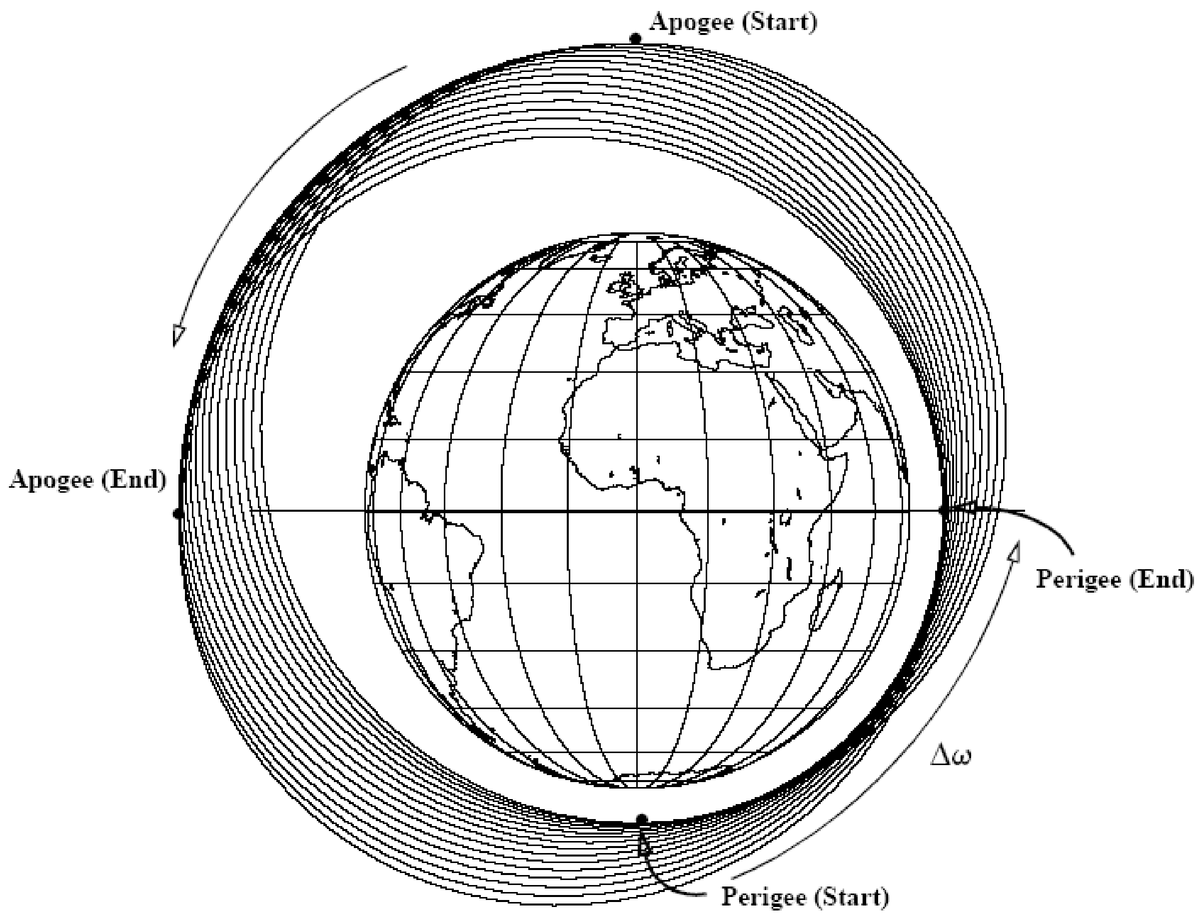
Nodal Regression



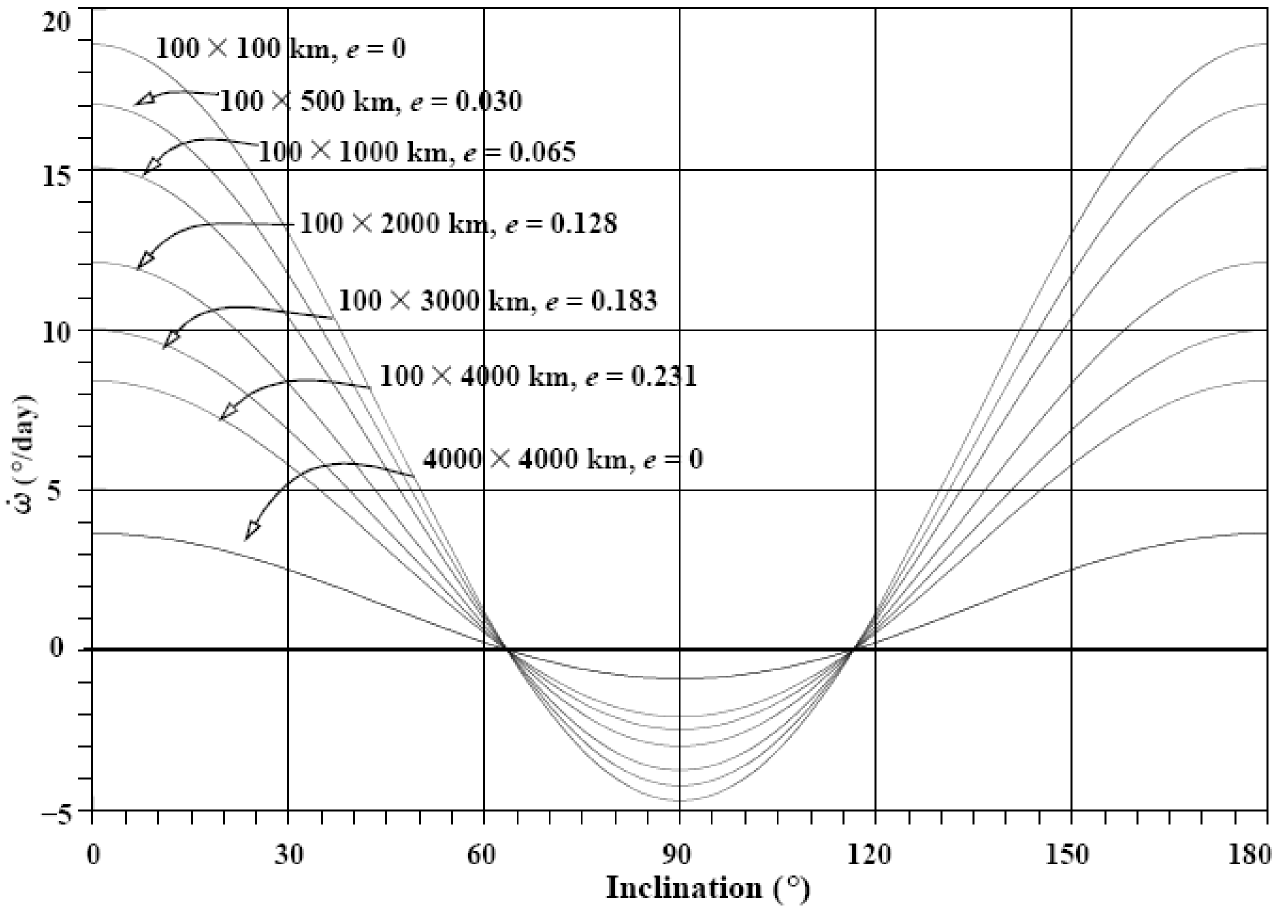
Nodal Regression



Apsidal Rotation



Apsidal Rotation



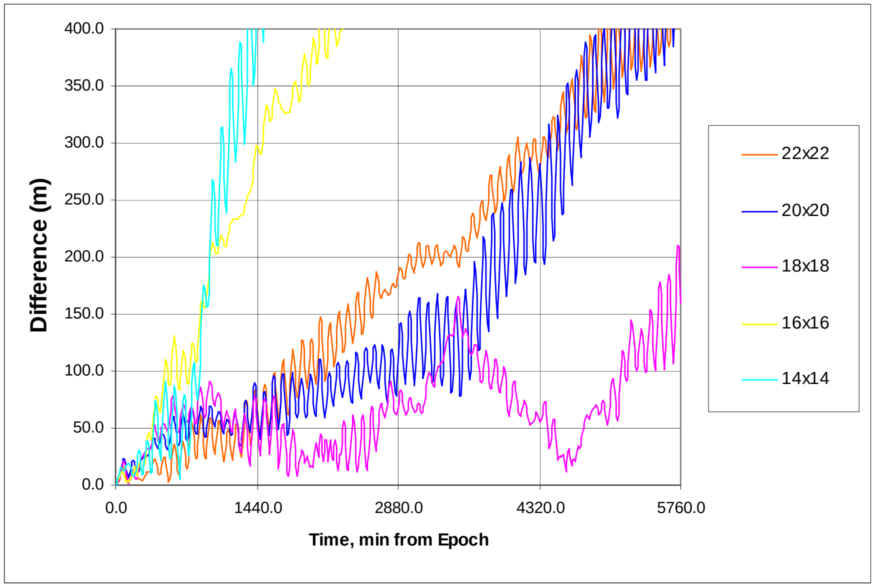
Gravitational Effects

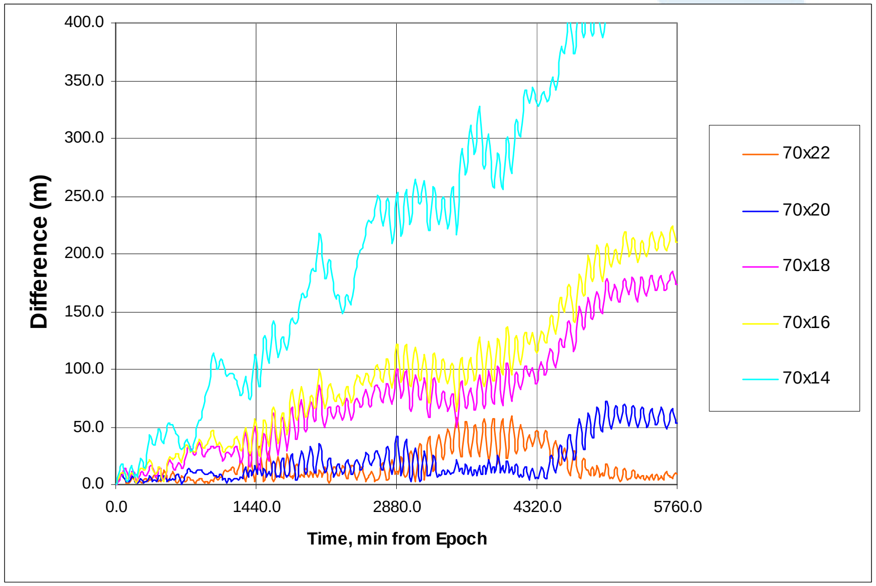
- Long ago when computers were slow…
- Gravitational modeling
  – Often square gravity field truncations
    - Appears the zonals contribute more

  – Point to take away:
    - Use “complete” field
    - Any truncations should include additional, if not all, zonal harmonics

Gravitational Modeling

- Satellite JERS (21867)
  - Comparison to 12x12 field
  - Note the variability over time
    - 22x22 vs 18x18 and 70x22 vs 70x18



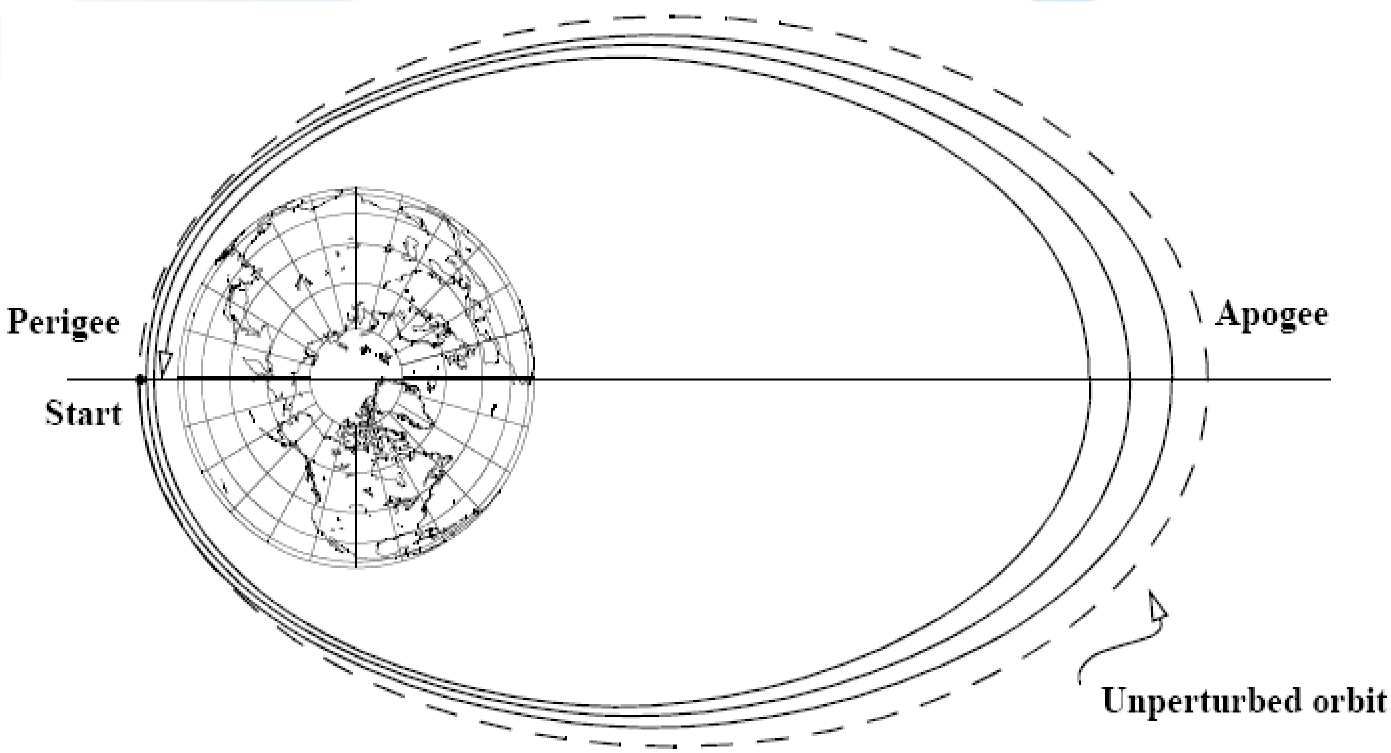


Atmospheric Drag

- Large force for near-Earth satellites
  - Very difficult to model
- Non-conservative force
  - Total kinetic and potential energy not constant
    - Heat, other losses through friction

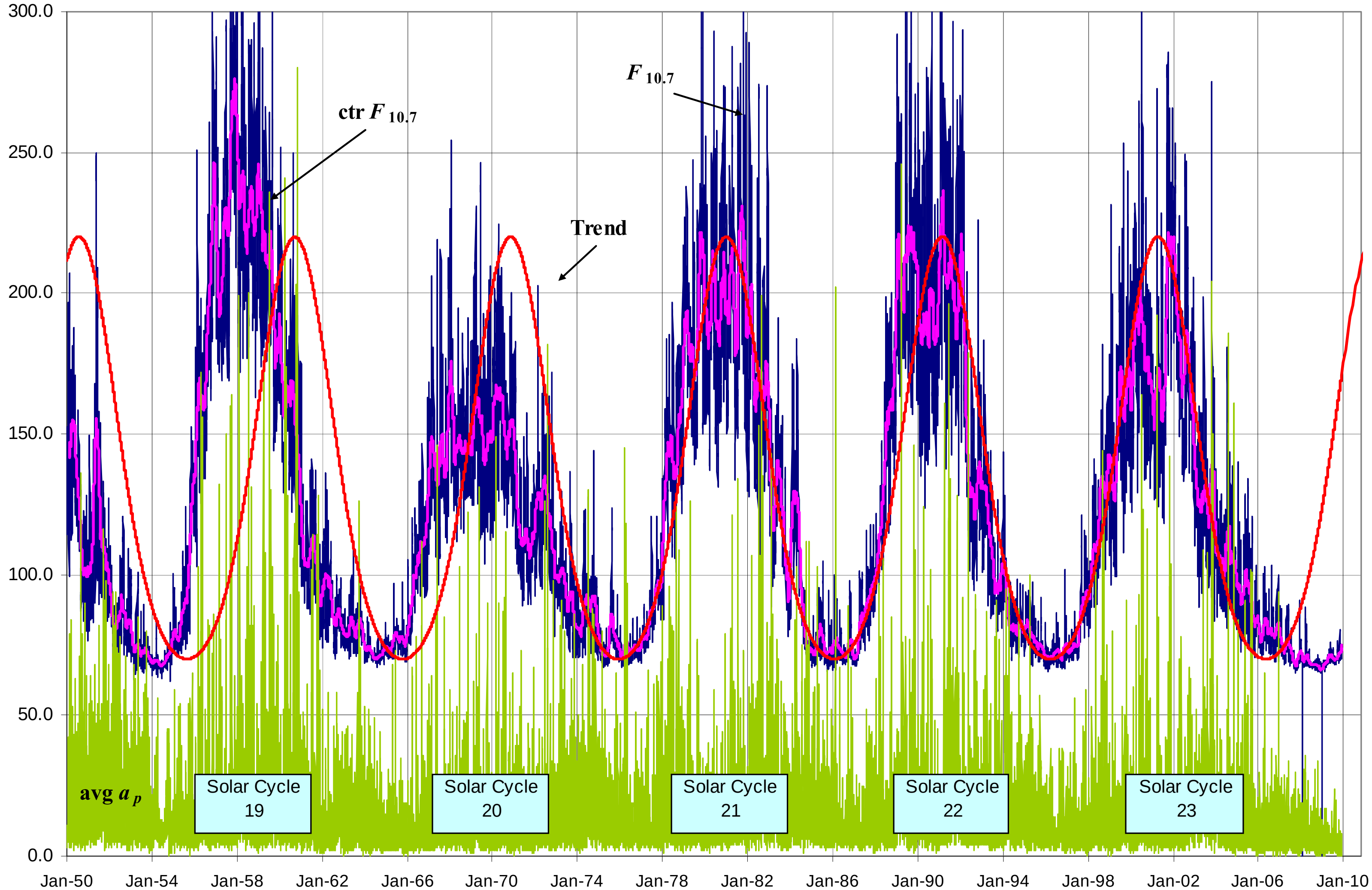
$$\vec{a}_{drag} = -\frac{1}{2}\rho\frac{c_D A}{m}v_{rel}^2\frac{\vec{v}_{rel}}{\left|\vec{v}_{rel}\right|}$$

Drag Effect on Orbits



Orbit tends to circularize

Available Data

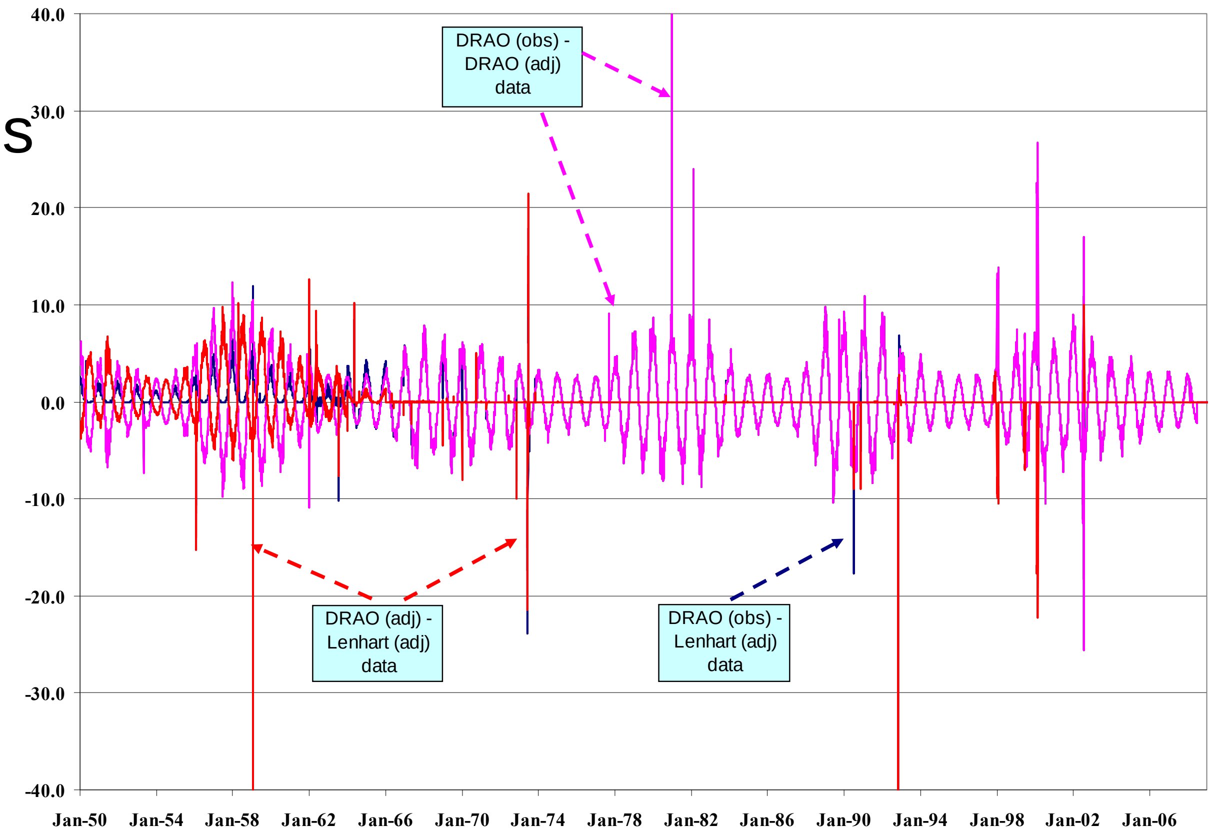


Space Weather – Predictions

- Lots of Variability
  - Constant $F_{10.7}$
    - Not very accurate
    - Never use 0.0!
  - Schatten
    - Varies with each solar cycle
  - Polynomial Trend
  - Matches several solar cycles
    - $F_{10.7}$ = 145 + 75*COS{ 0.001696 t + 0.35*SIN(0.001696 t)}
      - t is the number of days from Jan 1, 1981

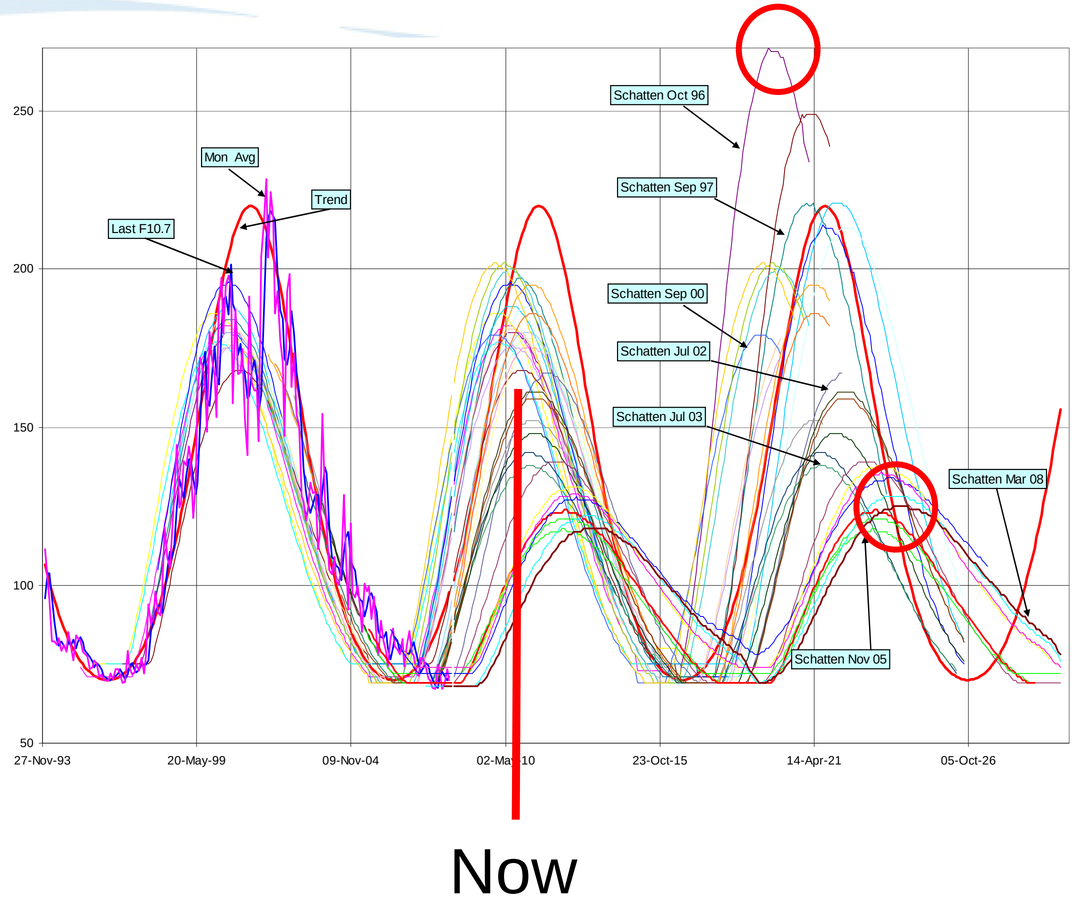
Observed vs Adjusted Solar Flux

- Data errors
  - Some inconsistencies
    - 10-40 SFU
  - Which does the model require?
    - MSIS
      - Observed
    - Others
      - Adjusted



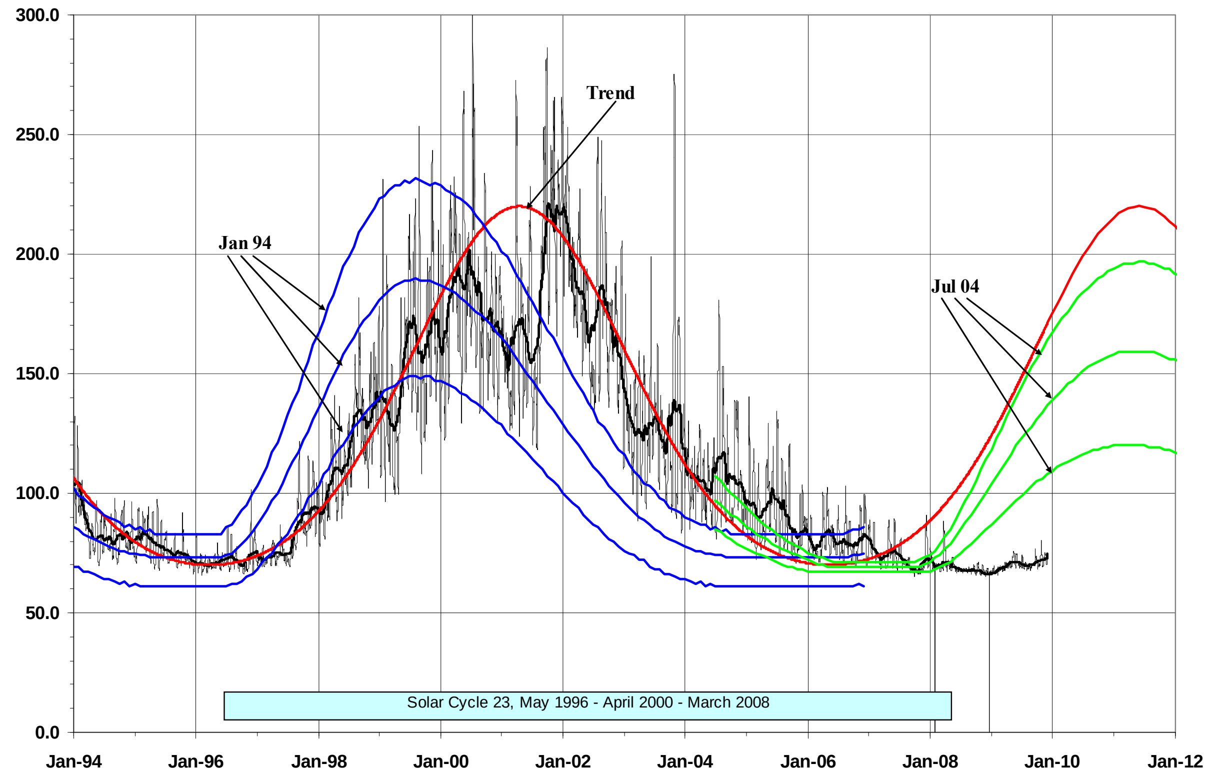
Solar Flux Predictions – Long Term

- Data differences
  - One solar cycle
    - ~150 SFU
  - Almost equal to the solar min-max difference!



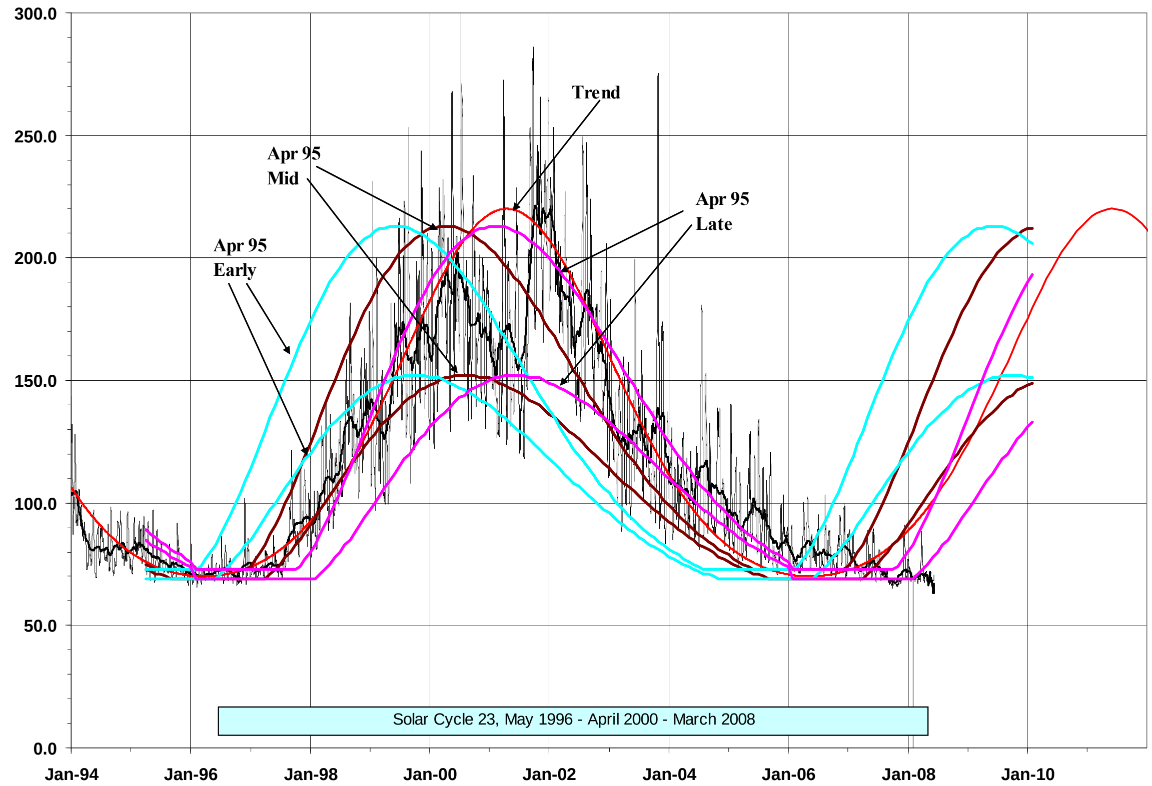
Solar Flux Predictions – Shorter Term

- Data differences
  - Min, Mid, and Max
    - 30-50 SFU
  - Note timing of Cycle is off



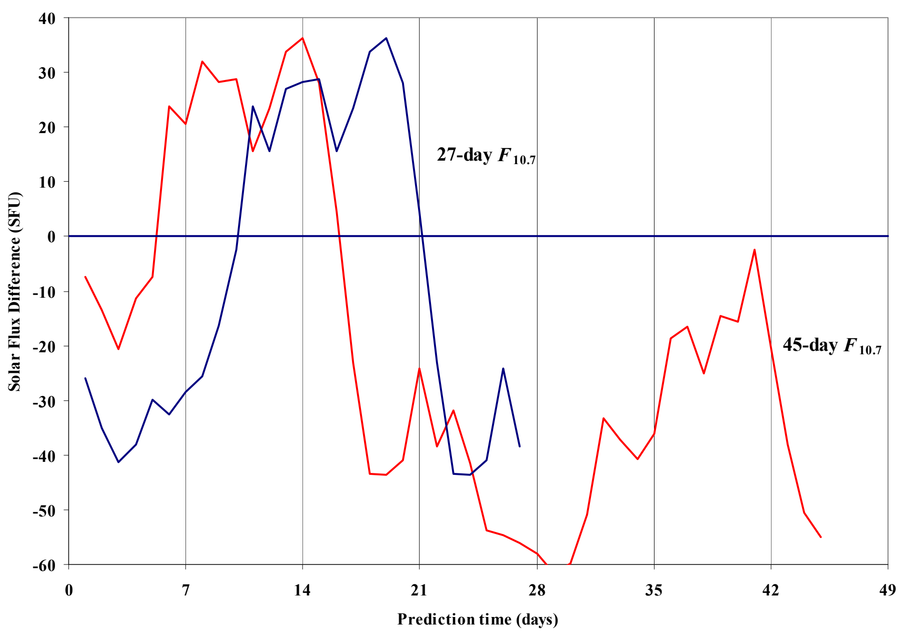
Solar Flux Predictions – Shorter Term

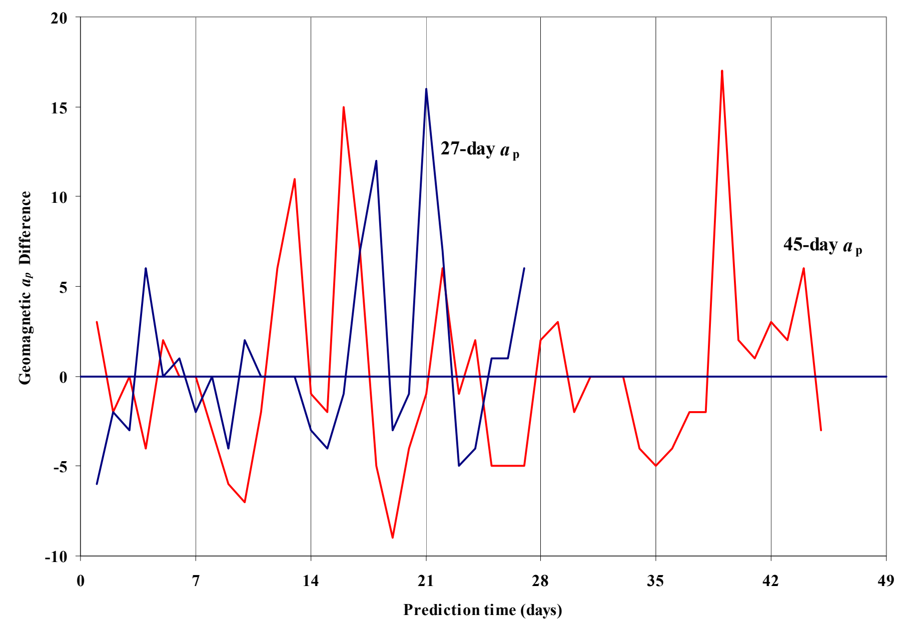
- Early, Mid, and Late
  - Also 30-50 SFU differences



Solar Flux Predictions – Short Term

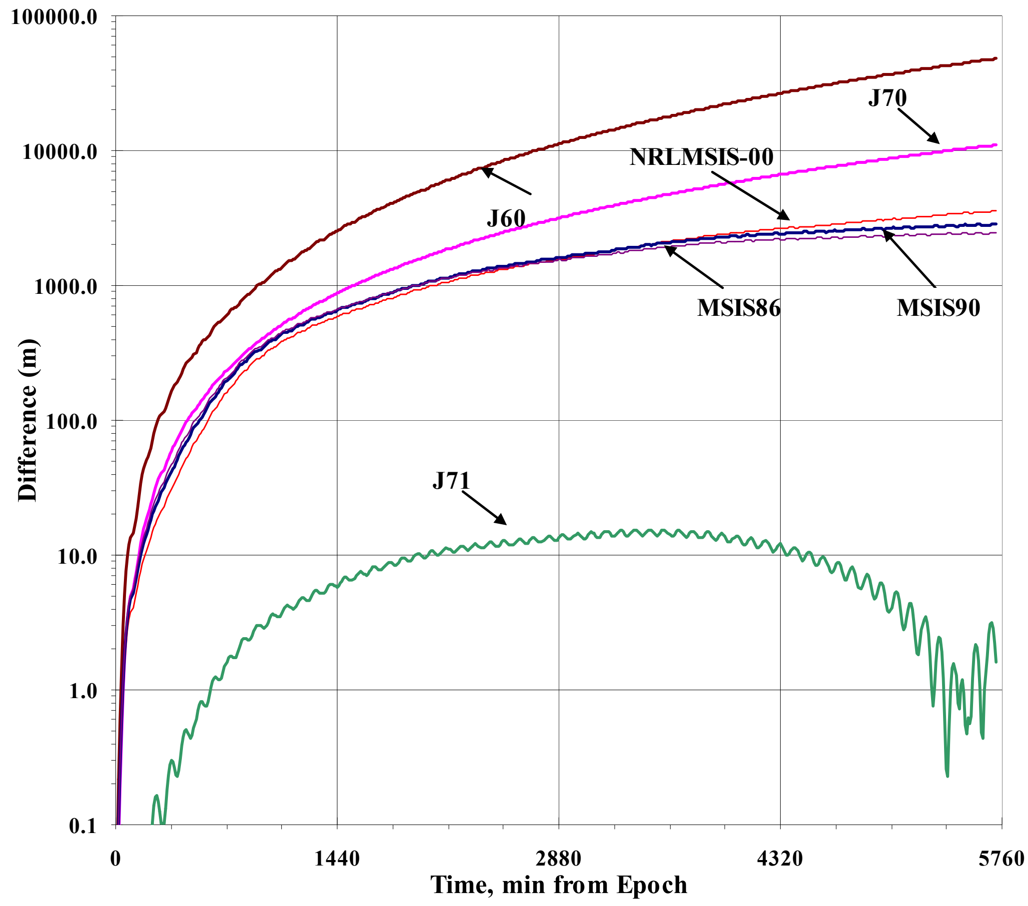
- NOAA Predictions
  - 27-day and 45-day ($F_{10.7}$ and a_p)
  - 3-day
    - 3-hourly K_p values off significantly as well





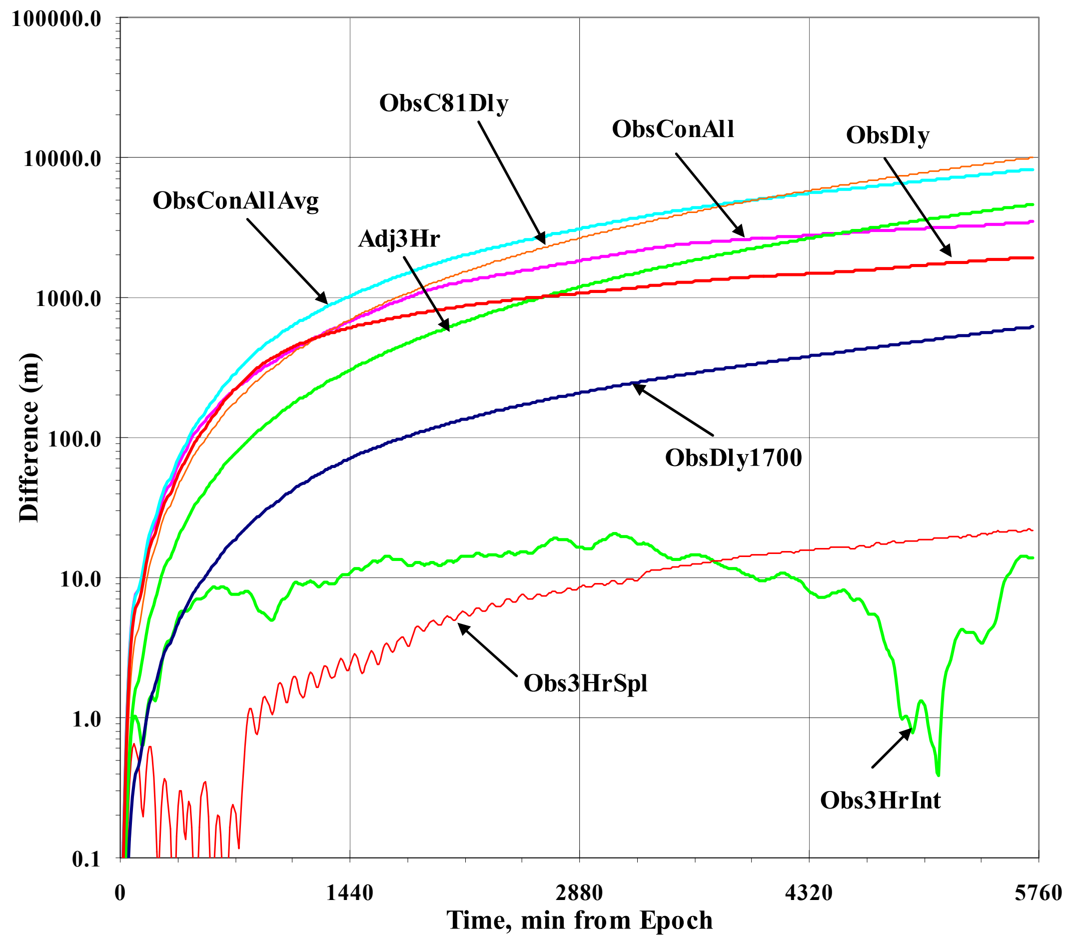
Simulated Sensitivity Analysis

- JERS sample orbit
  - Different atmospheric models
    - Baseline
      - Numerical propagation
      - Jacchia-Roberts
      - 3-hourly geomagnetic
  - Relative comparison only

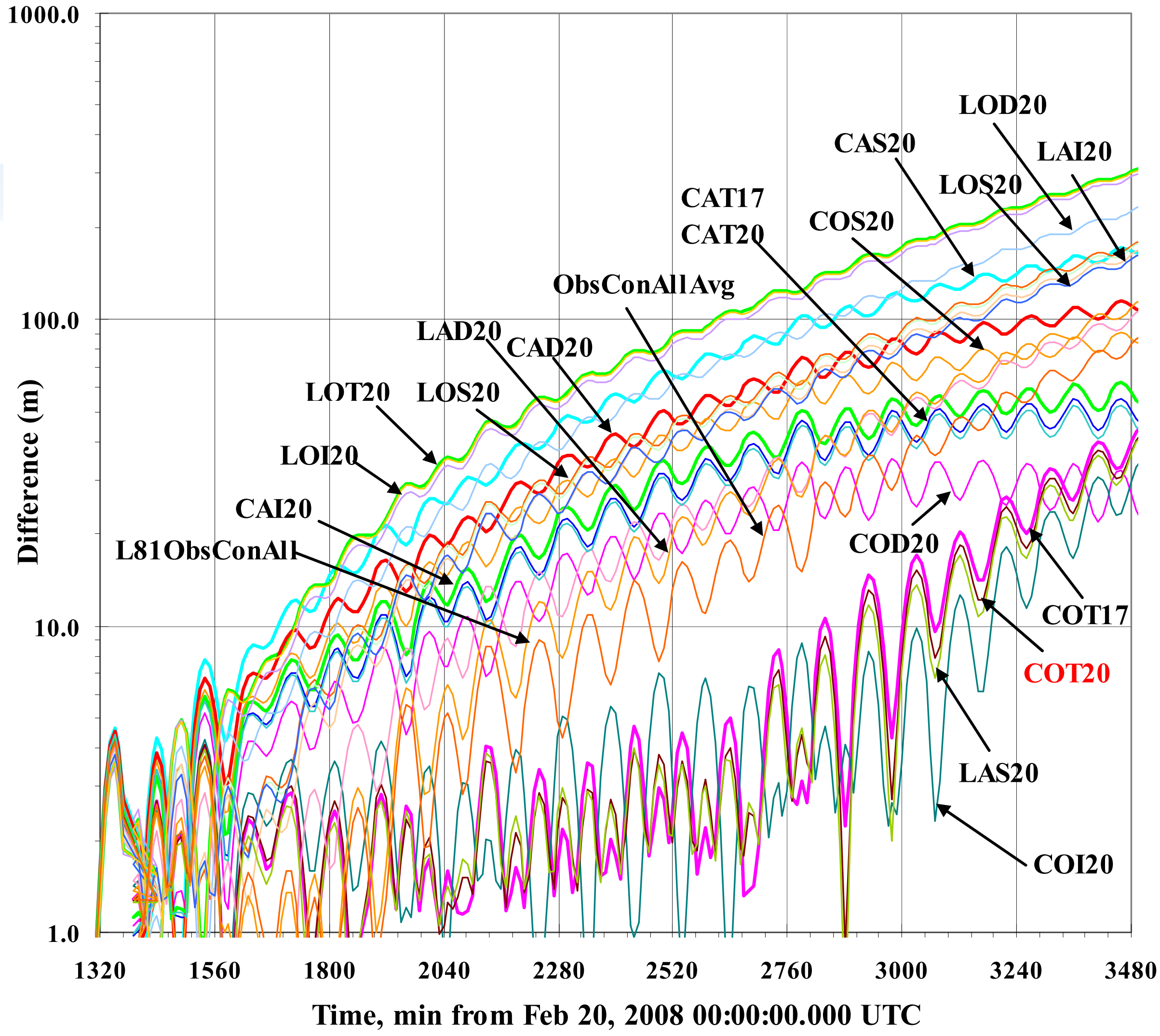


Simulated Sensitivity Analysis

- JERS sample orbit
  - Different treatment of the data
  - Baseline
    - Numerical Propagation
    - Jacchia-Roberts
    - 3-hourly geomagnetic

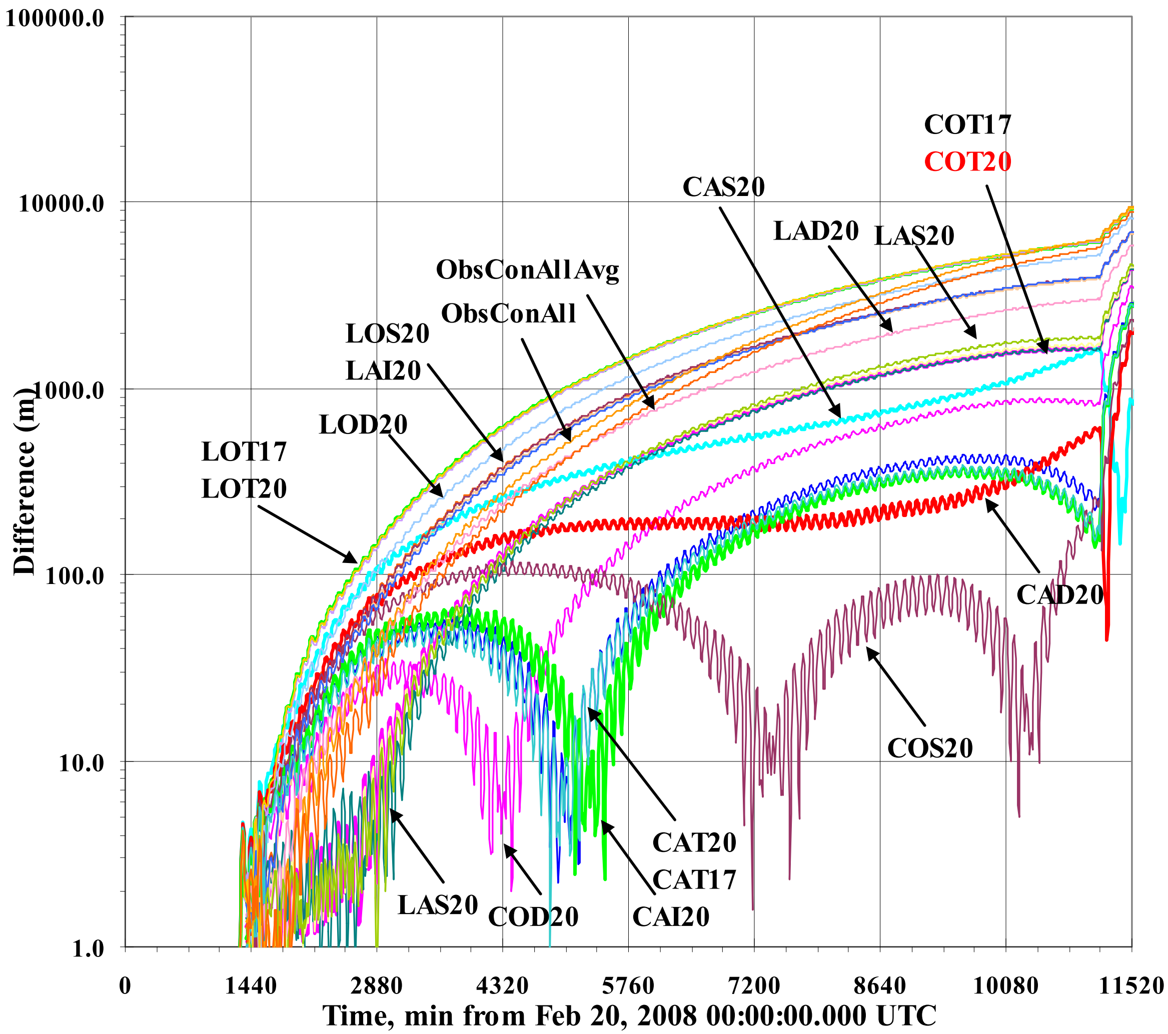


NRLMSISE-00 Results – Short Term

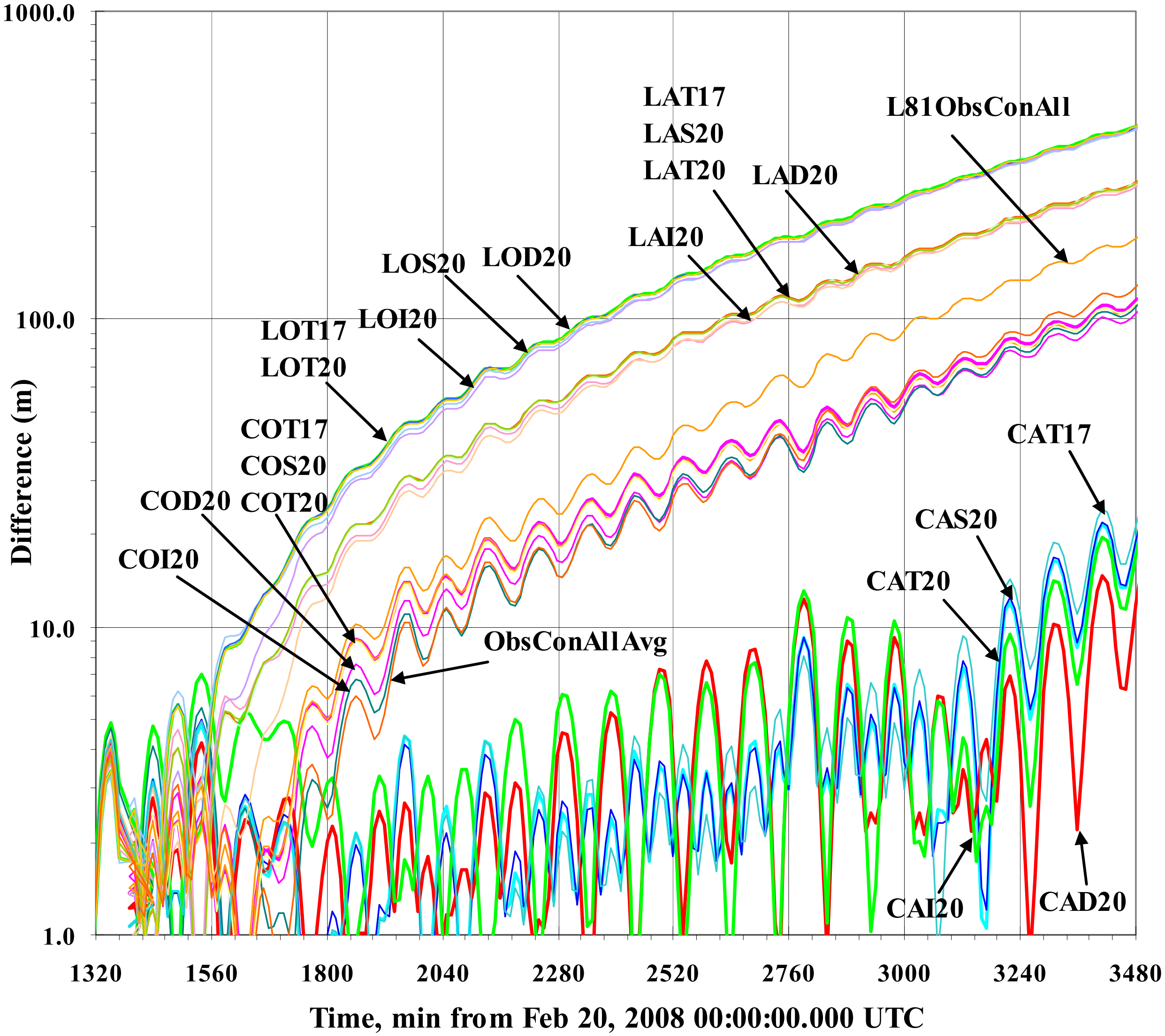


NRLMSISE-00 Results – Long Term

- Observations:
  - Model specifies observed
    - Adjusted performed well
  - Centered 81-day best
  - 20:00 UTC best
  - Spline interpolation very good
- No single best answer

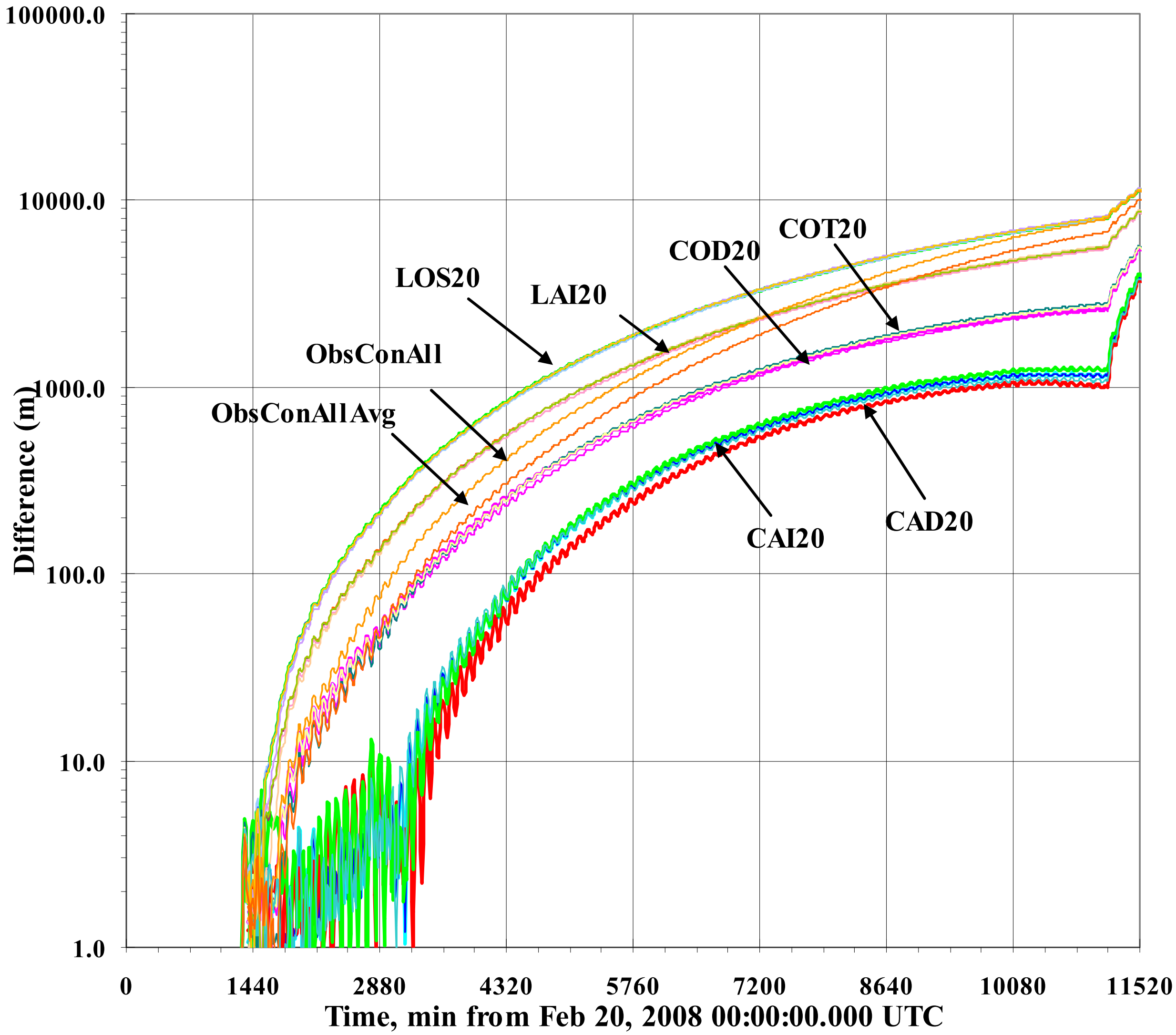


Jacchia-Roberts Results – Short Term



Jacchia-Roberts Results – Long Term

- Observations:
  - Adjusted performed well in all cases
  - Centered 81-day best
  - 20:00 UTC best
  - Daily geomagnetic very good, but all were close
- No single best answer



Atmospheric Drag Effects

- Atmospheric Drag
  - Large variations
    - Changing the atmospheric model
    - Changing how the input data is interpreted
      - $F_{10.7}$ at 2000 UTC
      - Last 81-day average $F_{10.7}$ vs. the central 81-day average
      - Using step functions for the atmospheric parameters vs interpolation
      - Many others (see AIAA and UC paper)
  - Point to take away:
    - 1-1000 km differences are possible
    - Unable to determine if from data interpretation or model differences

Third Body Forces

- Can affect GEO satellites strongly
- Conservative force (like gravity)

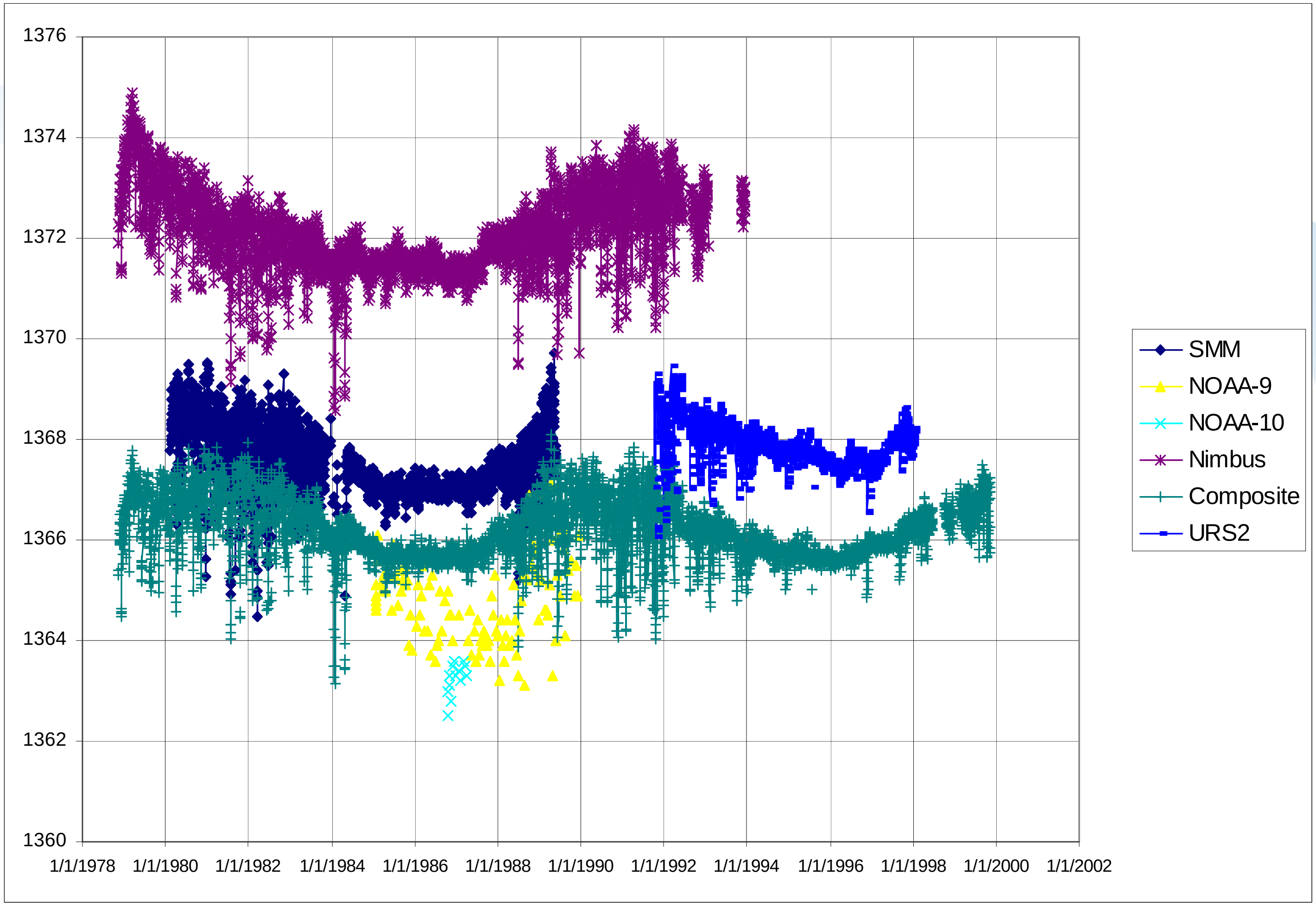
$$\boxed{\vec{a}_{3-body} = -\frac{G(m_{\oplus} + m_{sat})\vec{r}_{\oplus sat}}{r_{\oplus sat}^{3}} + Gm_{3}\left(\frac{\vec{r}_{sat3}}{r_{sat3}^{3}} - \frac{\vec{r}_{\oplus 3}}{r_{\oplus 3}^{3}}\right)}$$

Solar Radiation Pressure

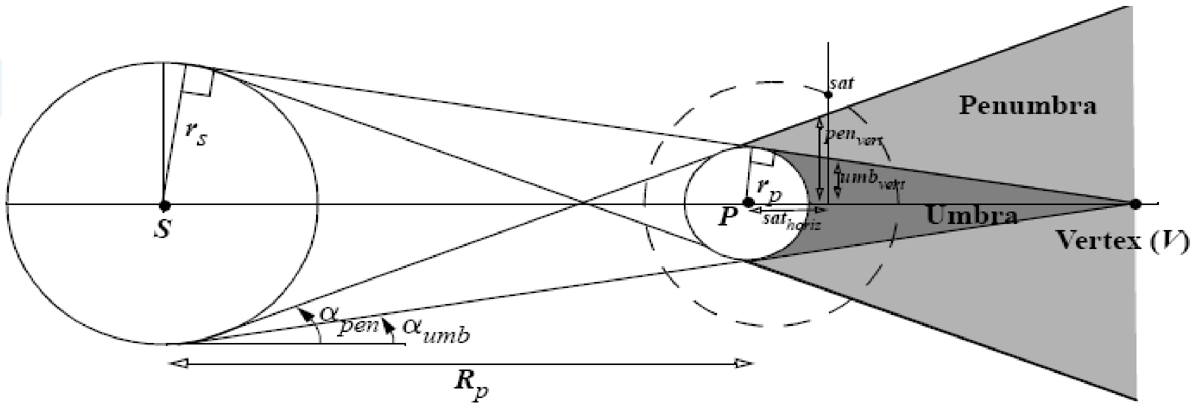
- Large effect for high altitude satellites (GPS, GEO, etc)
  - Non conservative force
- Shadowing by the Earth becomes very important
  - All satellite altitudes
- Solar Irradiance (p_{sr}) is difficult to measure accurately

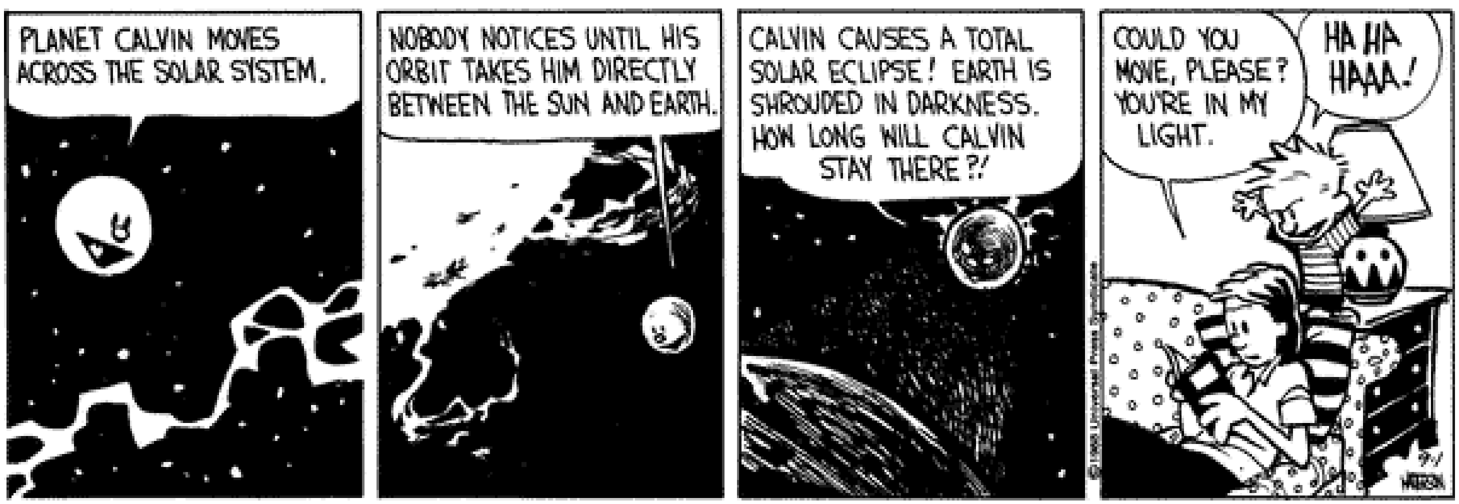
$$\vec{a}_{srp} = -p_{SR} \frac{c_R A_{Sun}}{m} \frac{\vec{r}_{sat-Sun}}{\left|\vec{r}_{sat-Sun}\right|}$$

Solar Irradiance (W/m^2)

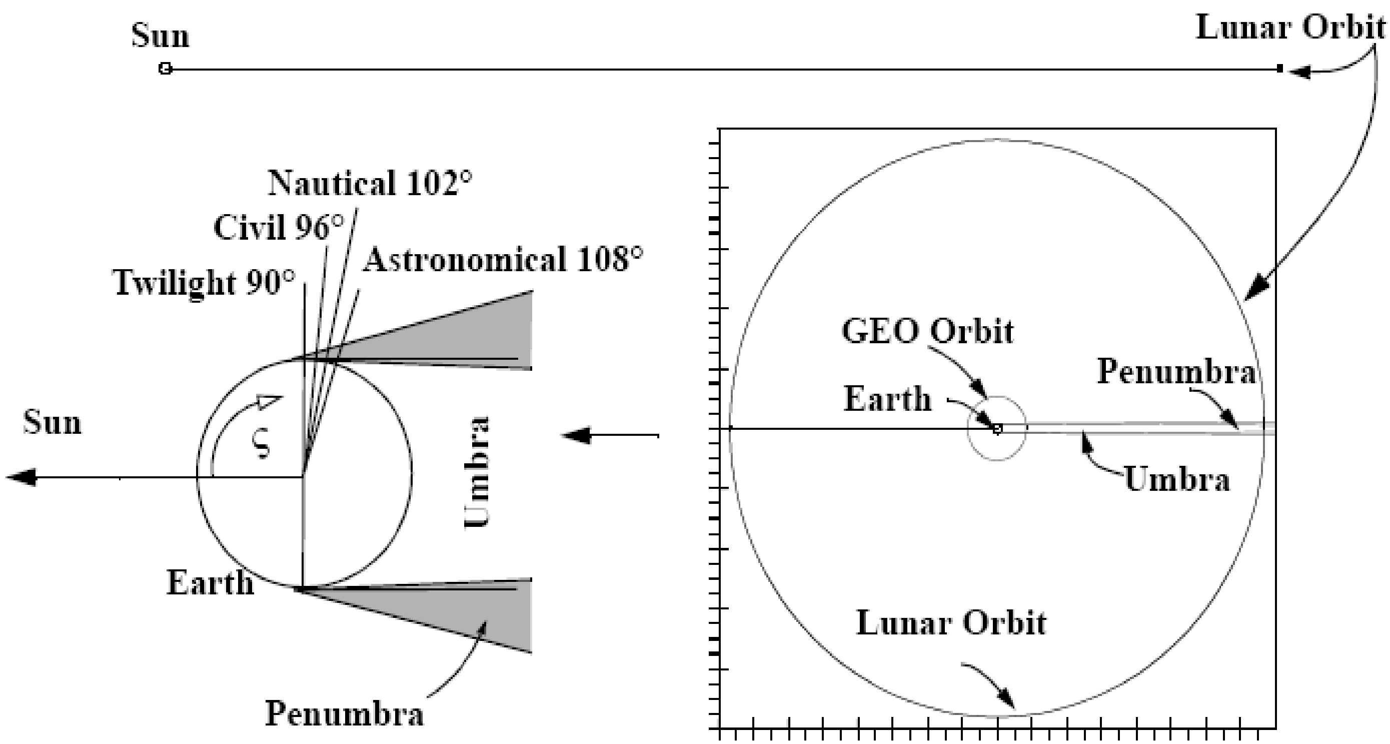


Earth Shadow Geometry



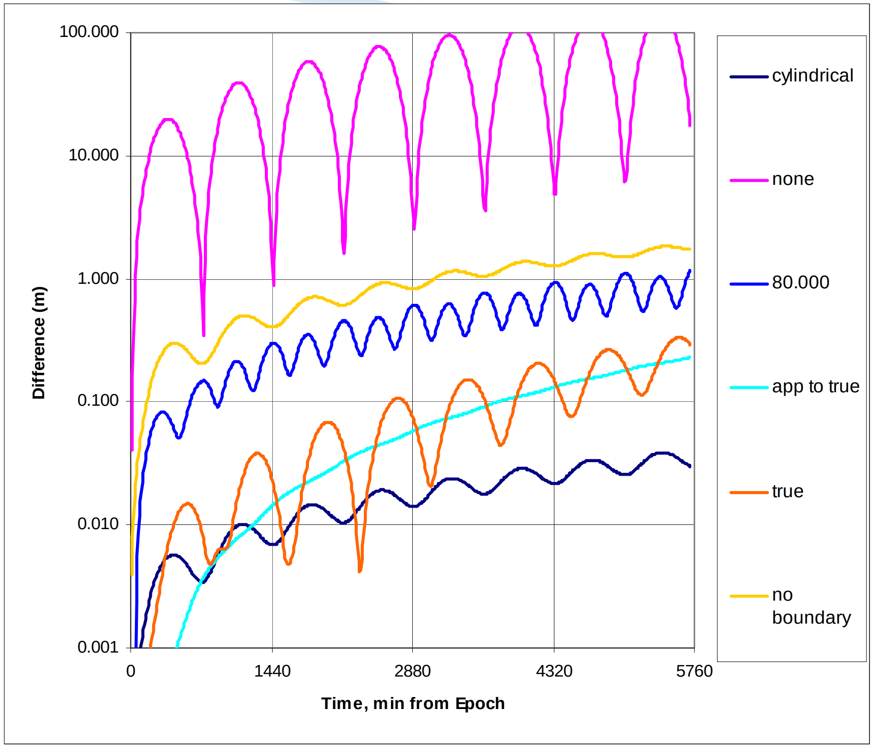


Earth Shadow Geometry



Solar Radiation Pressure Sensitivity Results

- Solar Radiation Pressure
  - Several variations shown
    - 26690 (GPS)
  - Notice max is ~100m
  - Definitions
    - Cylindrical
      - Defines shadow type
    - App to true
      - Acct for light travel from Sun to CB
    - True
      - Inst light from Sun
    - No Boundary
      - Change step size at penumbra/umbra
  - Point to take away
    - Relatively small effect
    - Some variations



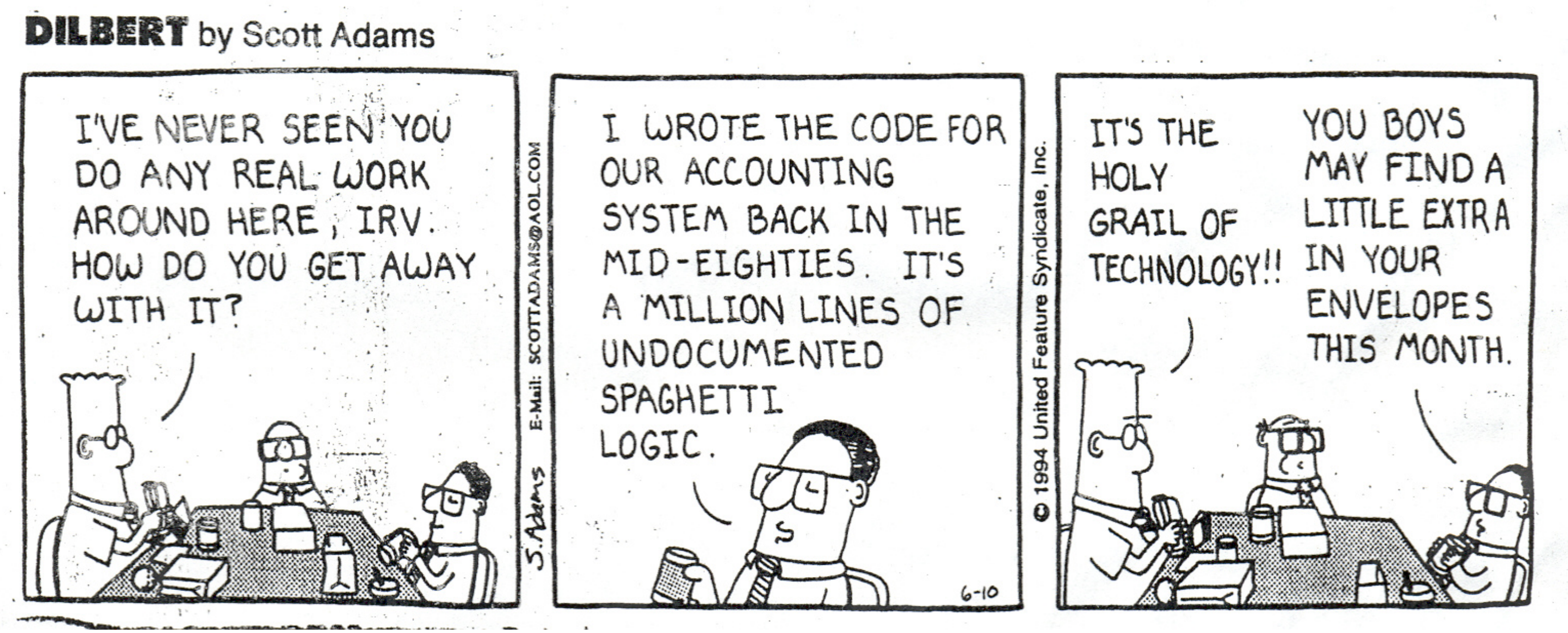
Special Perturbations

- Numerically integrate the equations of motion
  - Time consuming, but accurate

$$\vec{a} = \frac{\mu r}{r^3}\vec{r} + \vec{a}_{non-spherical} + \vec{a}_{drag} + \vec{a}_{3-body} + \vec{a}_{srp} + \vec{a}_{tides} + \vec{a}_{other}$$

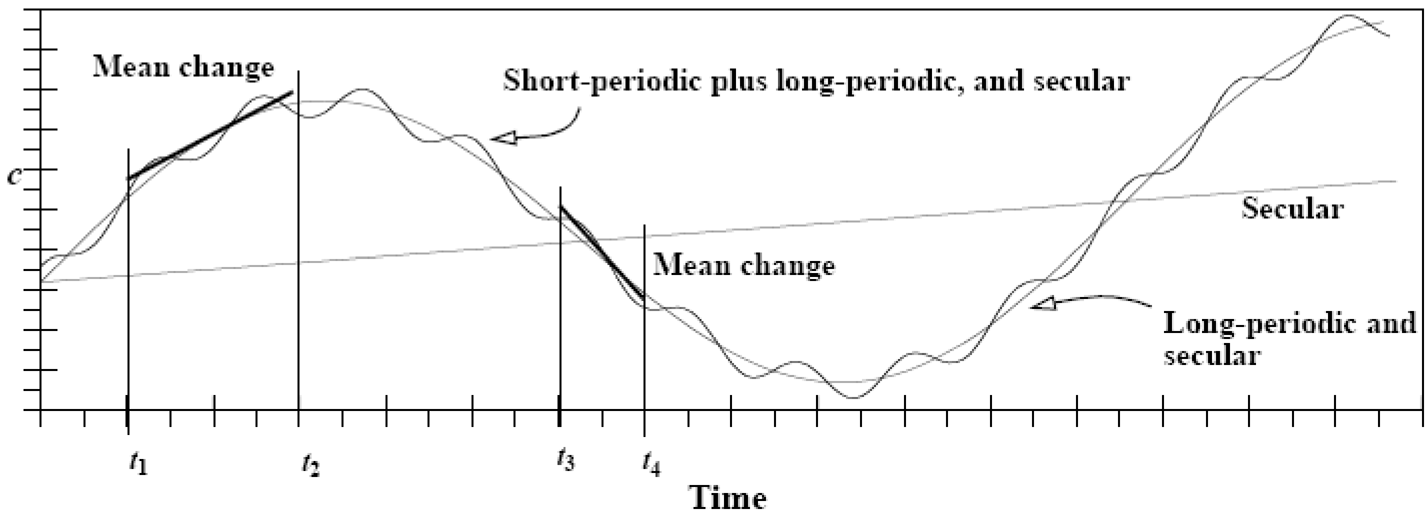
General Perturbations

- Truncate analytical expansions and solve directly
  - Large time steps
- Each approach is mathematically different
  - SGP4



Semianalytical

- Blend numerical and analytical
  - Analytically solve secular and long period components
  - Numerically integrate the small short period variations
- F



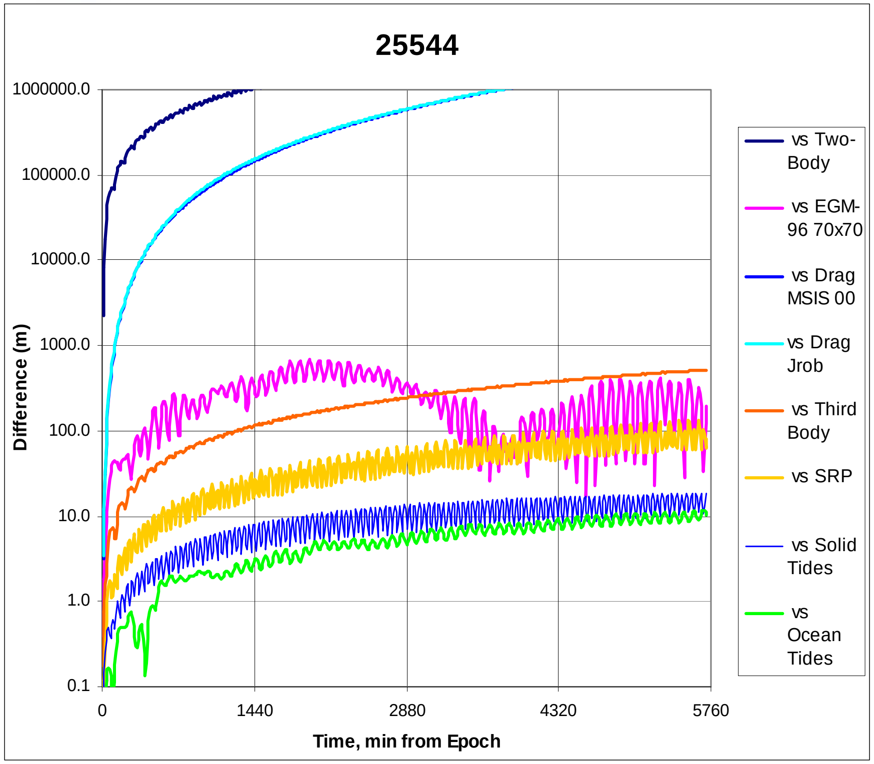
Force Model Sensitivity Results

- Force model contributions
  - Determine which forces contribute the largest effects
    - 12x12 gravity field is the baseline
  - Note
    - Gravity and Drag are largest contributors for LEO
    - 3rd body ~km effect for higher altitudes
  - Point to take away:
    - Trying to get the last cm from solid earth tides no good unless all other forces are at least that precise

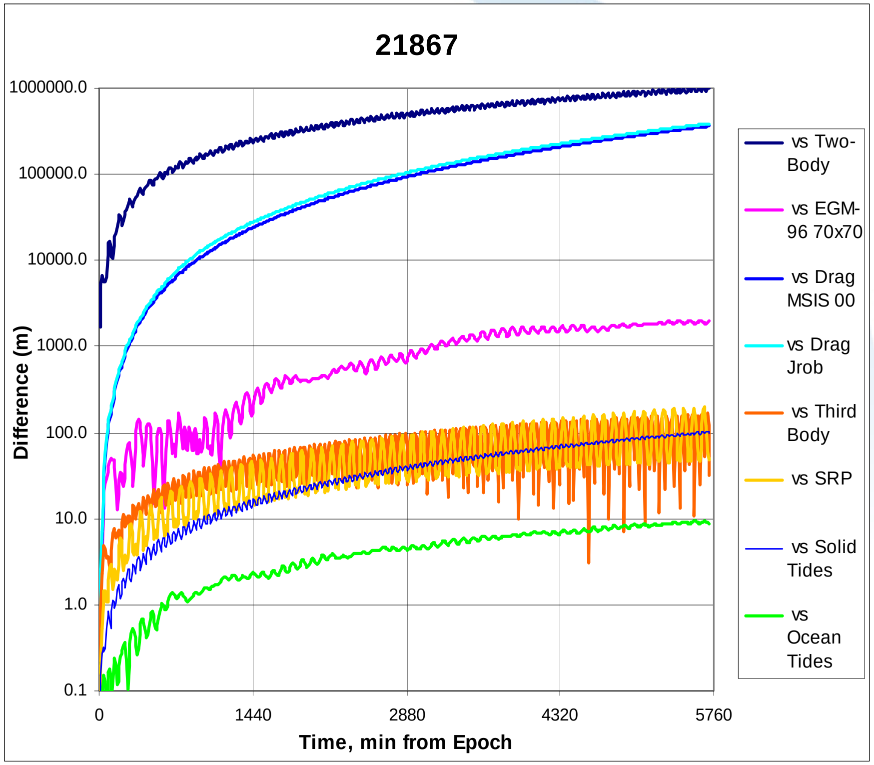
Force Model Contributions

- Low Earth Orbit

ISS



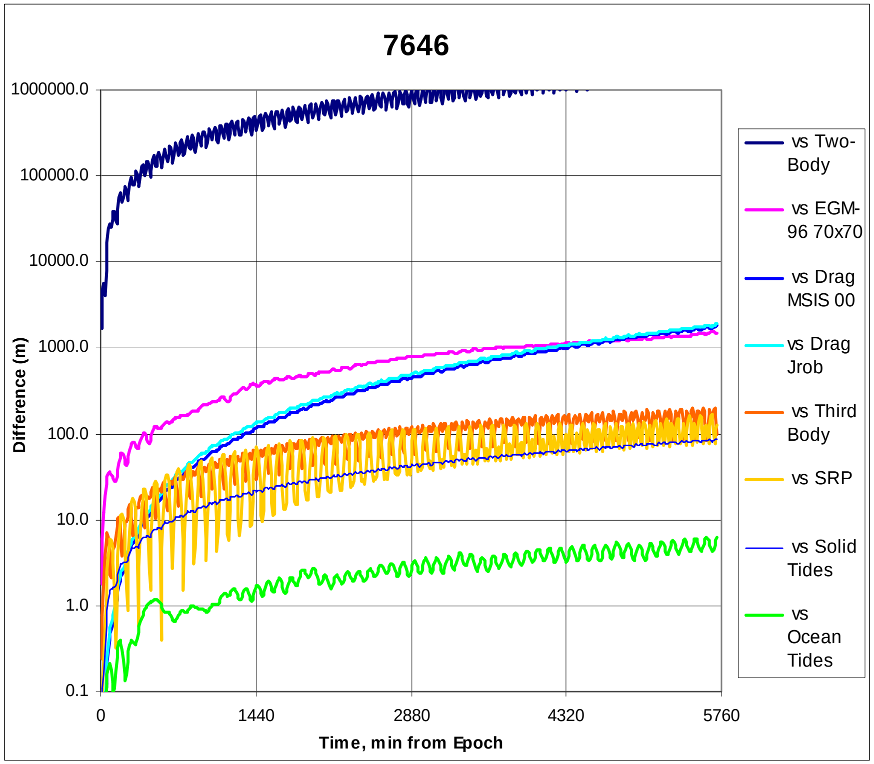
JERS



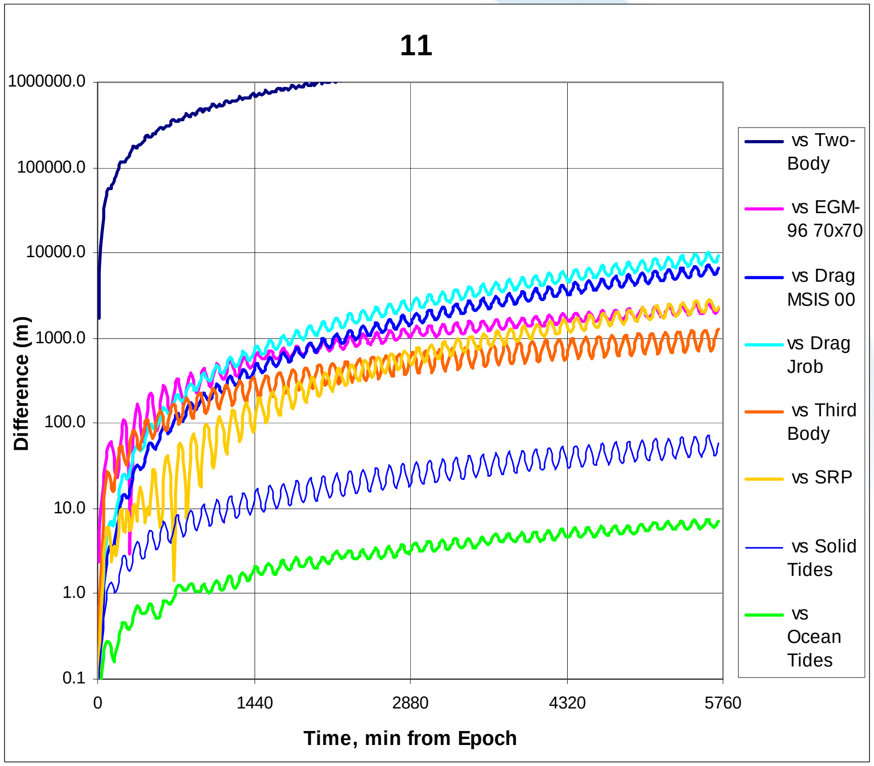
Force Model Contributions

- Low Earth Orbit

Starlette



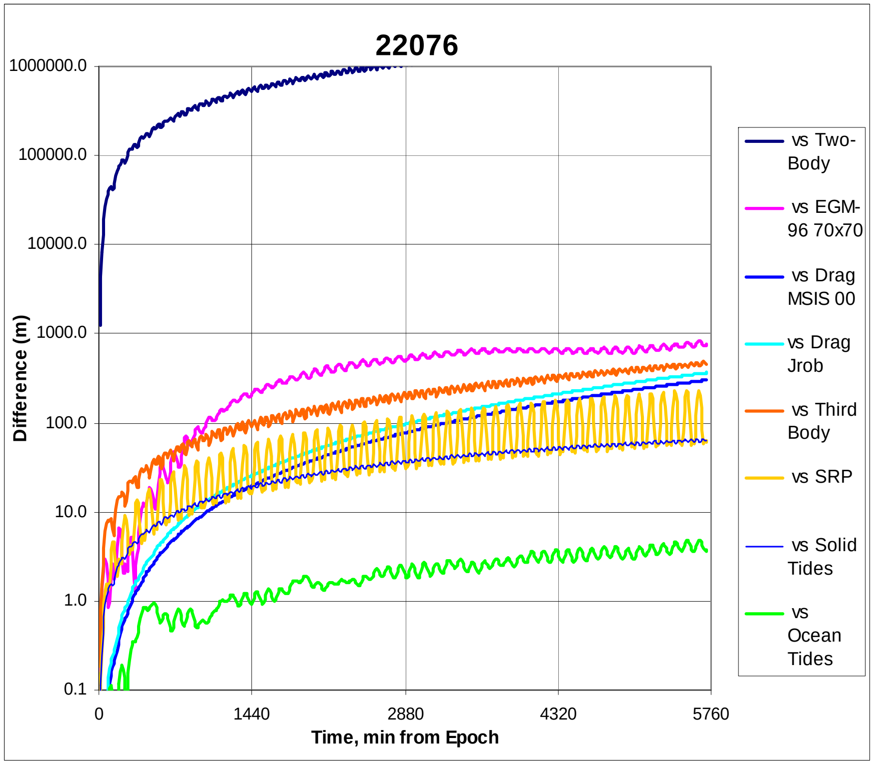
Vanguard II



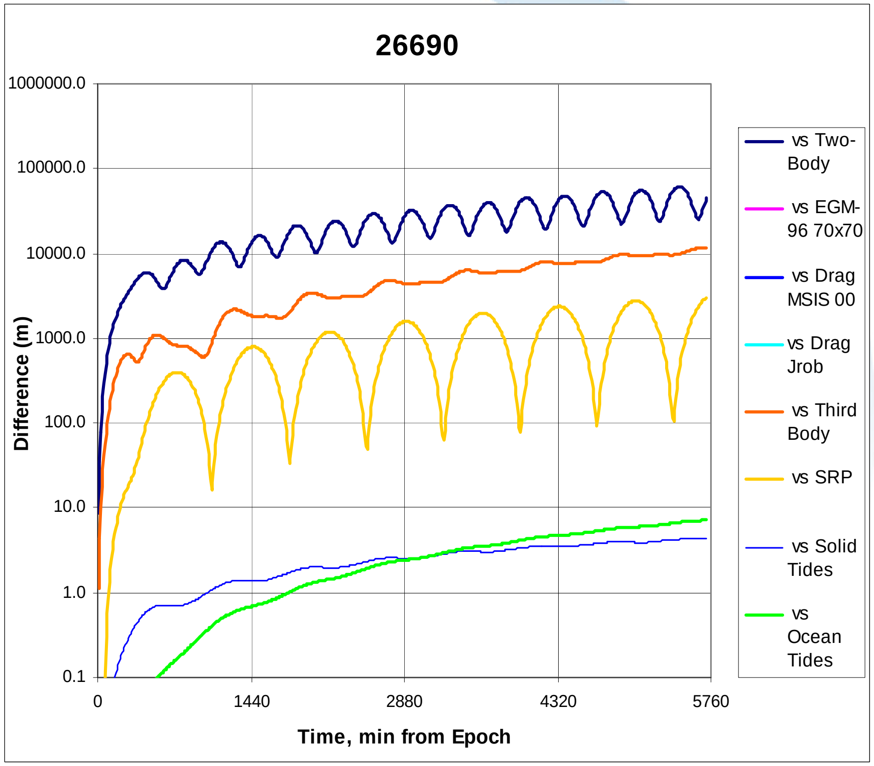
Force Model Contributions

- Low to Mid Earth Orbit

Topex



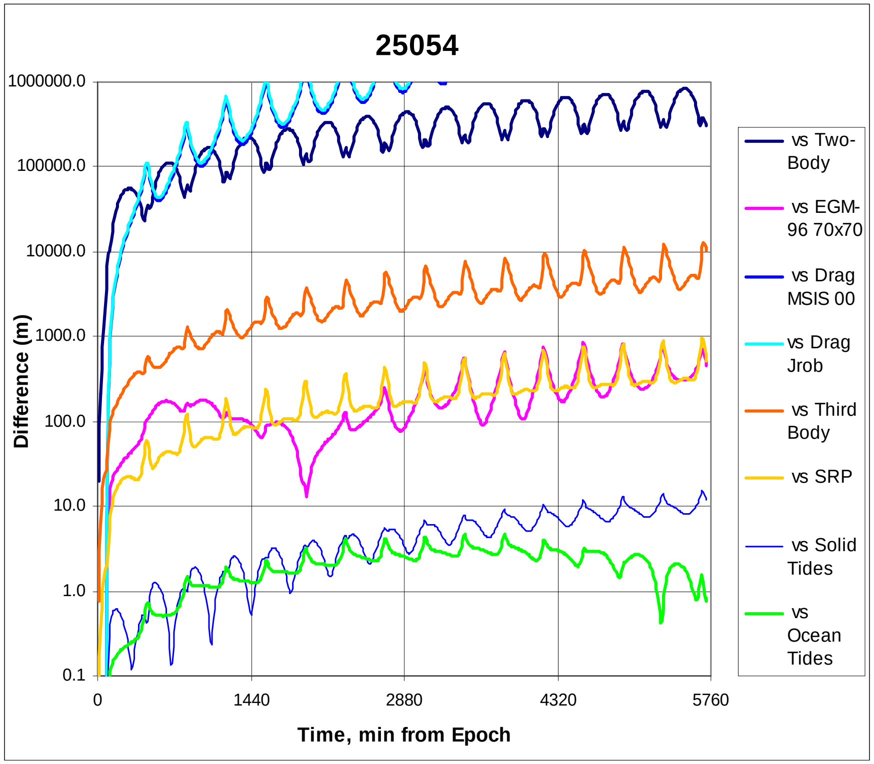
GPS



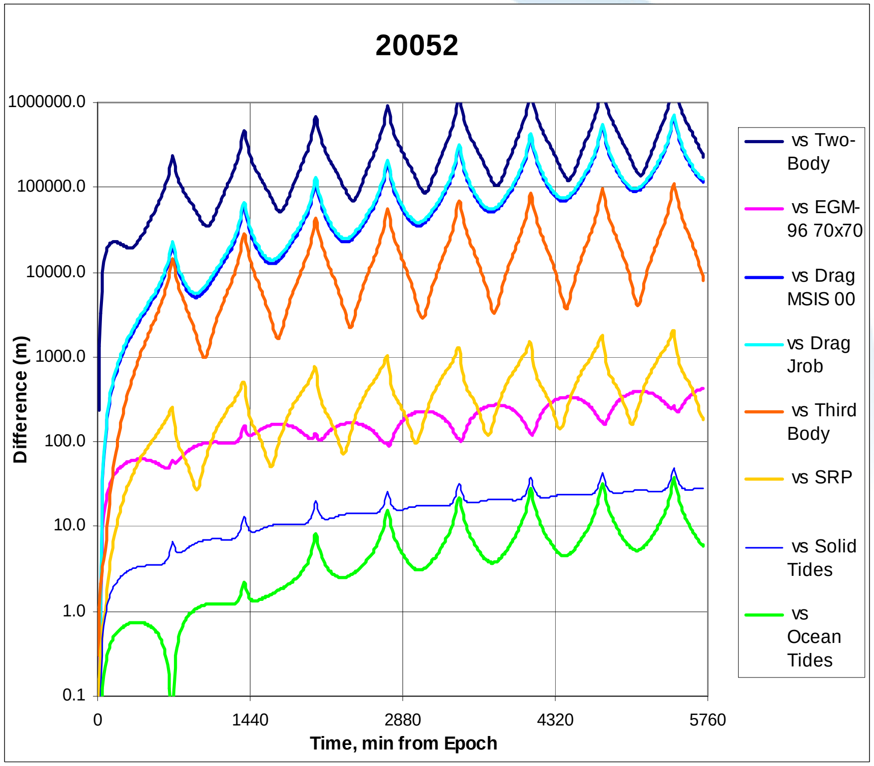
Force Model Contributions

- Mid Earth Orbit, eccentric

SL 12 RB



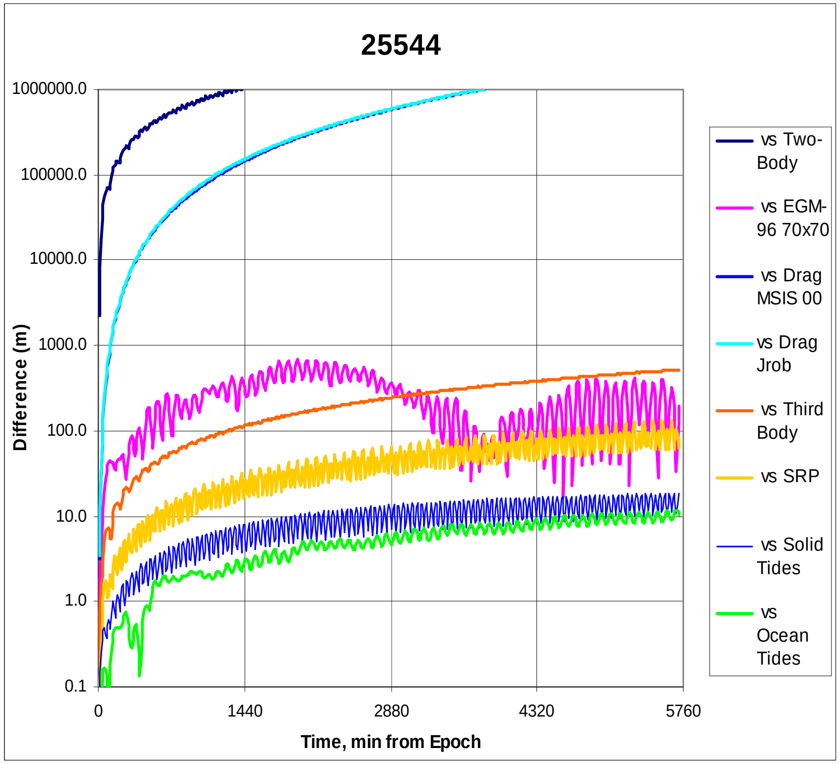
Molnyia



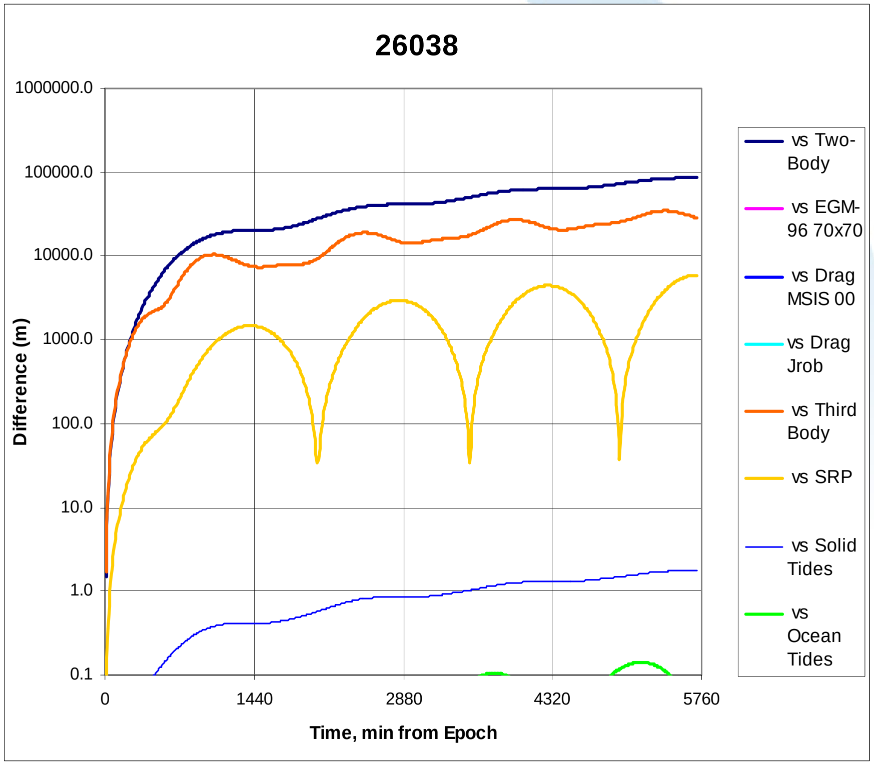
Force Model Contributions

- Low Earth and Geosynchronous Orbit

ISS (for comparison)



Galaxy 11



- Fundamental Concepts
- Newton
- Kepler
- Perturbations
- Orbit Determination
- Applications

Chapter 10

Terms

- Orbit Determination
  - Process of determining an orbit from observations
  - Also called Estimation
- Filtering
  - Determining the current state after each observation
- Smoothing
  - Improve previous state solutions using future data
  - Runs backwards

Terms

- Deterministic
  - Dynamics are known and can be calculated
  - Propagation
    - Assuming a specific set of force models
- Stochastic
  - Uses observations to correct for unknown or mis-modeled dynamics

Terms

- Least Squares
  - Minimizes the sum-square of the residuals
  - Depends on a fit span
    - Length of time to process a batch of observations
  - Often called Batch Least Squares (BLS)

Linear Least Squares Example

- Assume a mathematical model of motion

$$y = \alpha + \beta x$$

- Residuals defined as

$$r_i = y_{o_i} - y_{c_i} = y_{o_i} - (\alpha + \beta\, x_{o_i})$$

- Cost function (Jacobian)

$$J = \sum_{i=1}^{N} \overline{r_i}^2 = f(\alpha, \beta) = \sum_{i=1}^{N} (y_{o_i} - (\alpha + \beta x_{o_i}))^2$$

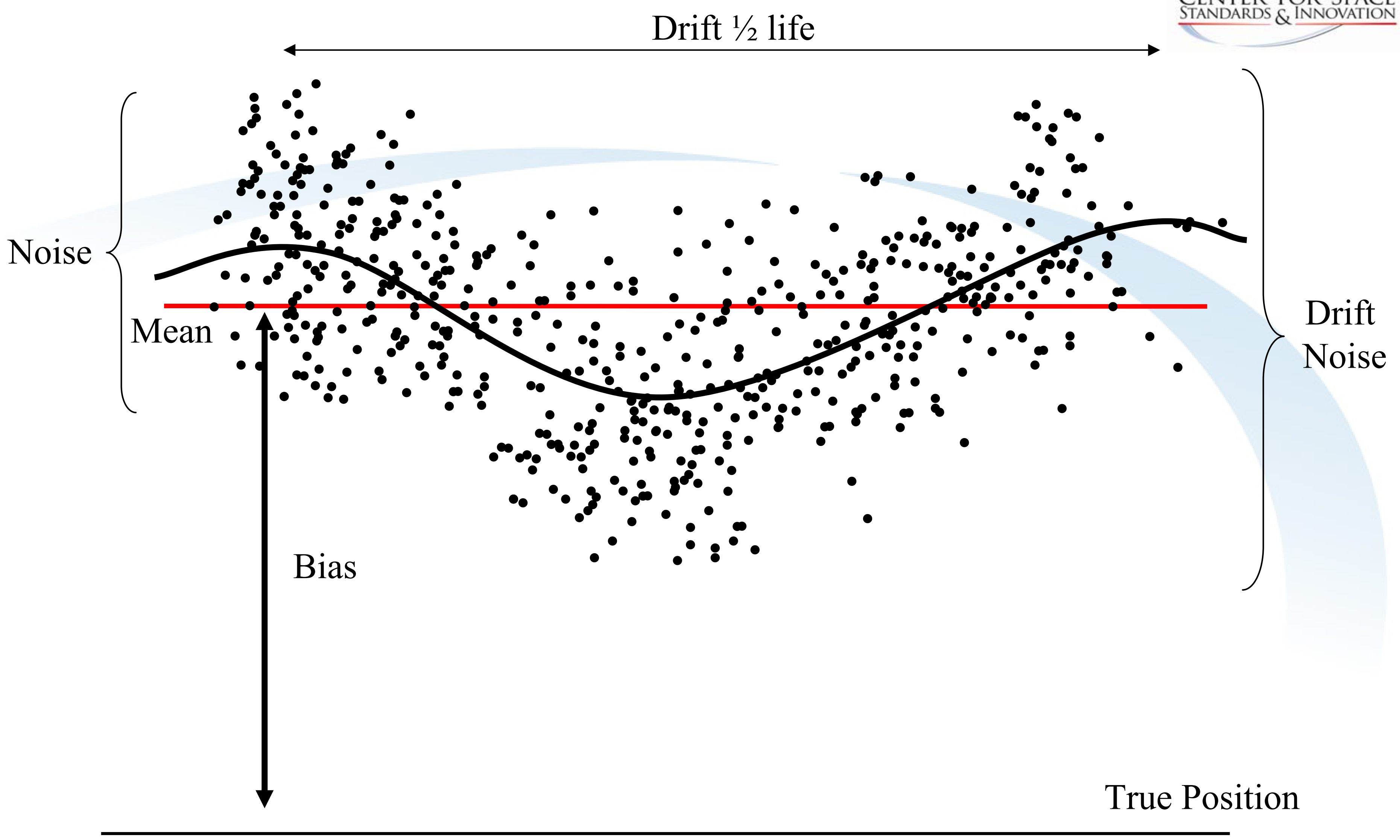
- Minimization of residuals

Linear Least Squares Example

- Matrix development

$$\underbrace{\begin{bmatrix} 1 & 1 & \dots & 1 \\ x_{o_1} & x_{o_2} & \dots & x_{o_N} \end{bmatrix}}_{A^T} \underbrace{\begin{bmatrix} 1 & x_{o_1} \\ 1 & x_{o_1} \\ \dots & \dots \\ 1 & x_{o_1} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \alpha \\ \beta \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 1 & 1 & \dots & 1 \\ x_{o_1} & x_{o_2} & \dots & x_{o_N} \end{bmatrix}}_{A^T} \underbrace{\begin{bmatrix} y_{o_1} \\ y_{o_2} \\ \vdots \\ y_{o_N} \end{bmatrix}}_{b}$$

- Normal Equation
  - $X = (A^T A)^{-1} A^T b$


Drift ½ life
Noise
Mean
Bias
Drift Noise
True Position

Statistical Concepts

- Dimensions and probability

Dimension	$z = 1\sigma$	2σ	3σ	4σ	
1	68.27	95.45	99.73	99.99	$erf(\frac{z}{\sqrt{2}})$
2	39.35	86.47	98.89	99.96	$1-\exp(\frac{-z^2}{2})$
3	19.87	73.85	97.07	99.89	$erf(\frac{z}{\sqrt{2}})-\sqrt{\frac{2}{\pi}}z\exp(\frac{-z^2}{2})$

Covariance Matrix

- Measure of uncertainty
- Grows with the satellite state propagation

$$\boldsymbol{P} = (\boldsymbol{A}^T\boldsymbol{W}\boldsymbol{A})^{-1}$$

  - $\boldsymbol{W}$ is weighting or sensor accuracies
  - $\boldsymbol{A}$ is partial derivative matrix
- Correlation Coefficients
  - Off diagonal terms
- Eigenvalues
  - Indicates each axis of the ellipsoid

LS Applied to Satellites: Overview

Initial Orbit Determination

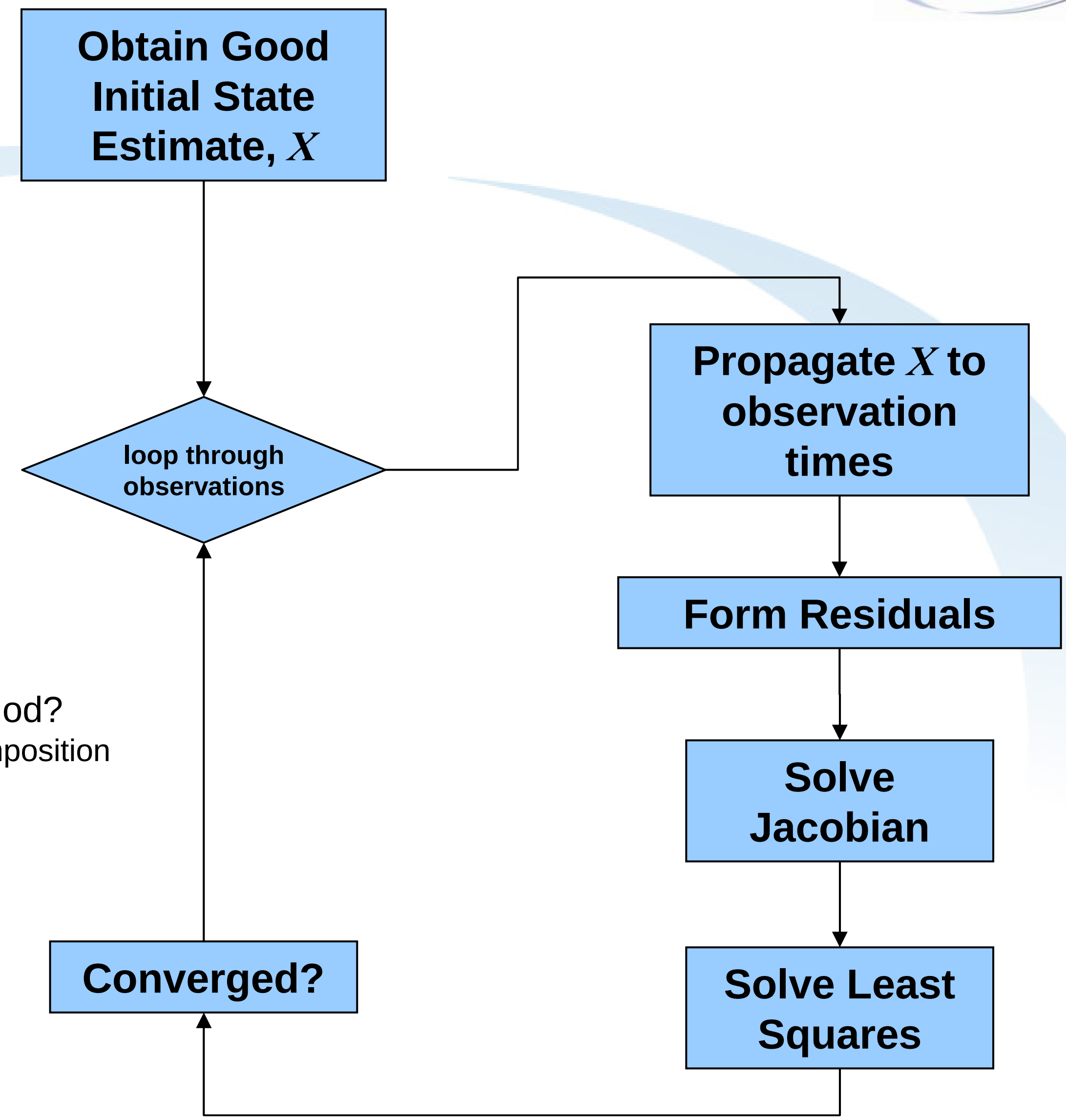
How good?
- Radius of Curvature

What state representation?
- Equinoctial, Keplerian, other

Orbit Determination

How to solve for Jacobian?
- Analytical, finite differencing

Least Squares Solution method?
- Classical, Single Value Decomposition



LS Algorithm: Matrix Inverse Approach

- FOR $i = 1$ to the number of observations (N)
  - Propagate (SGP4, HPOP) nominal state to time of observation (TEME, ICRF)
  - Find the slant range vector, sensor to the propagated state in the topocentric (SEZ) coordinate system
  - Determine nominal observations from the SEZ vector
  - Find the $\boldsymbol{b}$ matrix as observed – nominal observations
  - Form the $\boldsymbol{A}$ matrix
    - Finite (or central) differences
    - Analytical partials
      - $\boldsymbol{H}$, Partials depending on observation type
      - $\boldsymbol{\Phi}$, Partials for state transition matrix.
  - Accumulate $\boldsymbol{A}^T\boldsymbol{W}\boldsymbol{A}$ and $\boldsymbol{A}^T\boldsymbol{W}\boldsymbol{b}$
- END FOR
- Find $\boldsymbol{P} = (\boldsymbol{A}^T\boldsymbol{W}\boldsymbol{A})^{-1}$ using Gauss-Jordan elimination (LU decomposition and back-substitution)
- Solve $\delta\boldsymbol{x} = \boldsymbol{P}\,\boldsymbol{A}^T\boldsymbol{W}\underline{\boldsymbol{b}}$
- Check RMS for convergence
- Update state $\boldsymbol{X} = \boldsymbol{X} + \delta\boldsymbol{x}$
- Repeat if not converged using updated state

Sequential Batch Least Squares

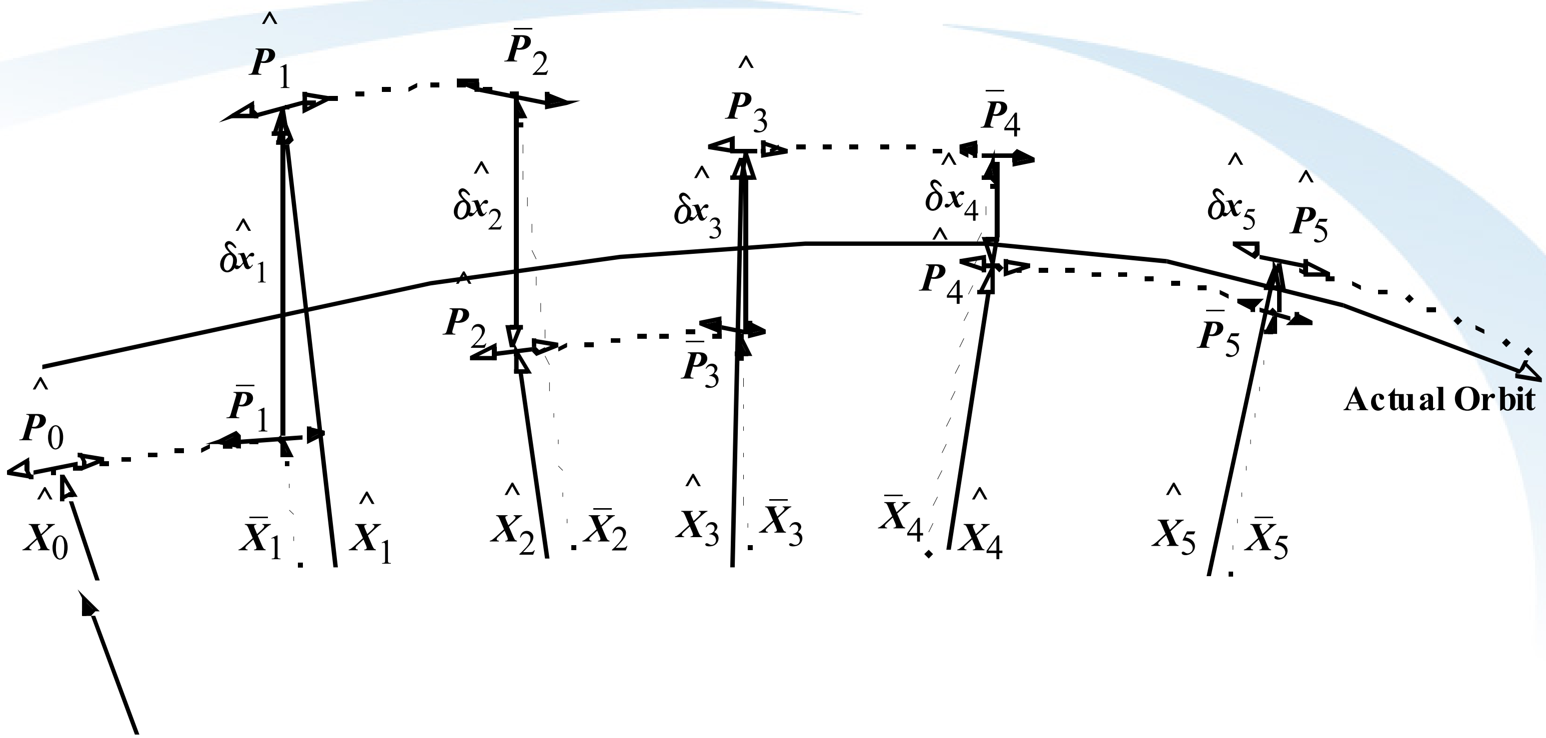
- Process additional observations
  - Use previous results
- Bayes Theorem

- Normal Equation
  - This is for "k" previously determined obs
  - "$k + n$" new obs

$$\delta x(0 \mid k+n) = (A_{new}^T W_{new} A_{new} + \hat{P}_k^{-1})^{-1} (A_{new}^T W_{new} \tilde{b}_{new} + A_k^T W_k \tilde{b}_k)$$

$$\hat{P}_{k+n} = \hat{P}(0 \mid k+n) = (A_{new}^T W_{new} A_{new} + \hat{P}_k^{-1})^{-1}$$

Extended Kalman Filter



Extended Kalman Filter

at each obs time $H_{k+1} = \frac{\partial z}{\partial \hat{X}_{k+1}}$

Prediction

$$\bar{X}(t_{k+1}, t_k) = \int_{t_k}^{t_{k+1}} \dot{\bar{X}}_{t_k} dt + \bar{X}_{t_k}$$ Predicted State

$$F = \frac{\partial \dot{\hat{X}}_{t_{k+1}}}{\partial \hat{X}_{t_{k+1}}}$$

$$\dot{\Phi}(t_{k+1}, t_k) = F(t)\Phi(t_{k+1}, t_k)$$

$$\delta \bar{x}_{k+1} = 0$$ Predicted State Error

$$\bar{P}_{k+1} = \Phi \bar{P}_k \Phi^T + Q$$ Predicted Error Covariance

Update

$$\tilde{b}_{k+1} = z - H_{k+1}\bar{X}_{k+1}$$

$$K_{k+1} = \bar{P}_{k+1} H_{k+1}^T [H_{k+1}\bar{P}_{k+1}H_{k+1}^T + R]^{-1}$$ Kalman Gain

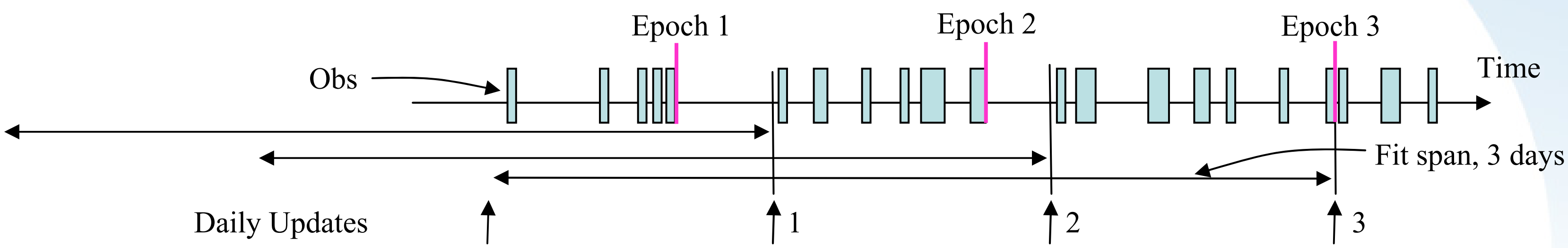
$$\delta \hat{x}_{k+1} = \delta \bar{x}_{k+1} + K_{k+1}\tilde{b}_{k+1}$$ State Error Estimate

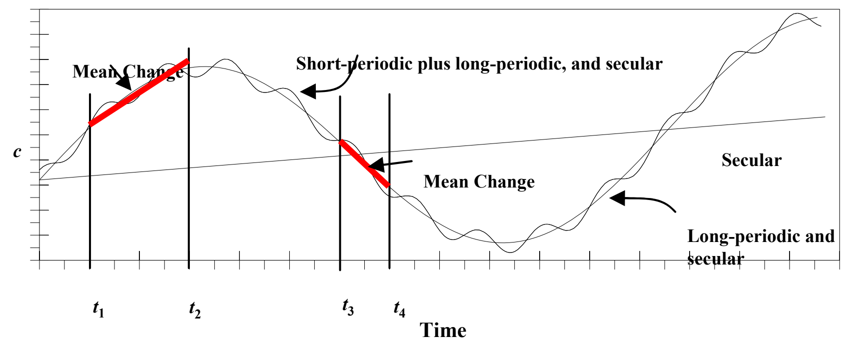
$$\hat{P}_{k+1} = \bar{P}_{k+1} - K_{k+1}H_{k+1}\bar{P}_{k+1}$$ Error Covariance Estimate

$$\hat{X}_{k+1} = \bar{X}_{k+1} + \delta \hat{x}_{k+1}$$ State Estimate

Averaging and Fit Spans

- Obs are taken periodically
- Updates often occur at regular intervals
- Least Squares approaches “average” data collected for a “batch” of time – the Fit Span





- Fundamental Concepts
- Newton
- Kepler
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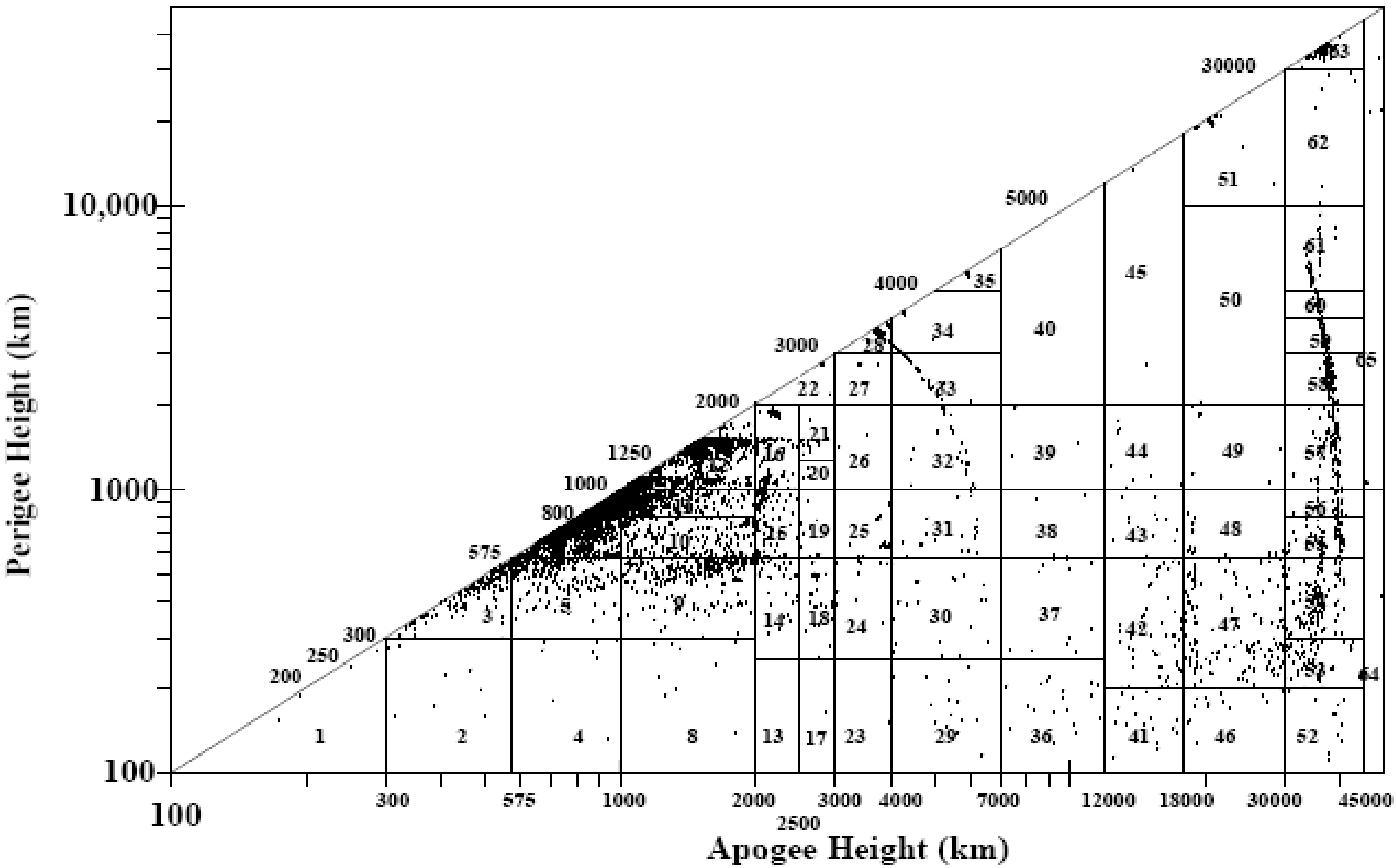
- Applications

Chapter 11

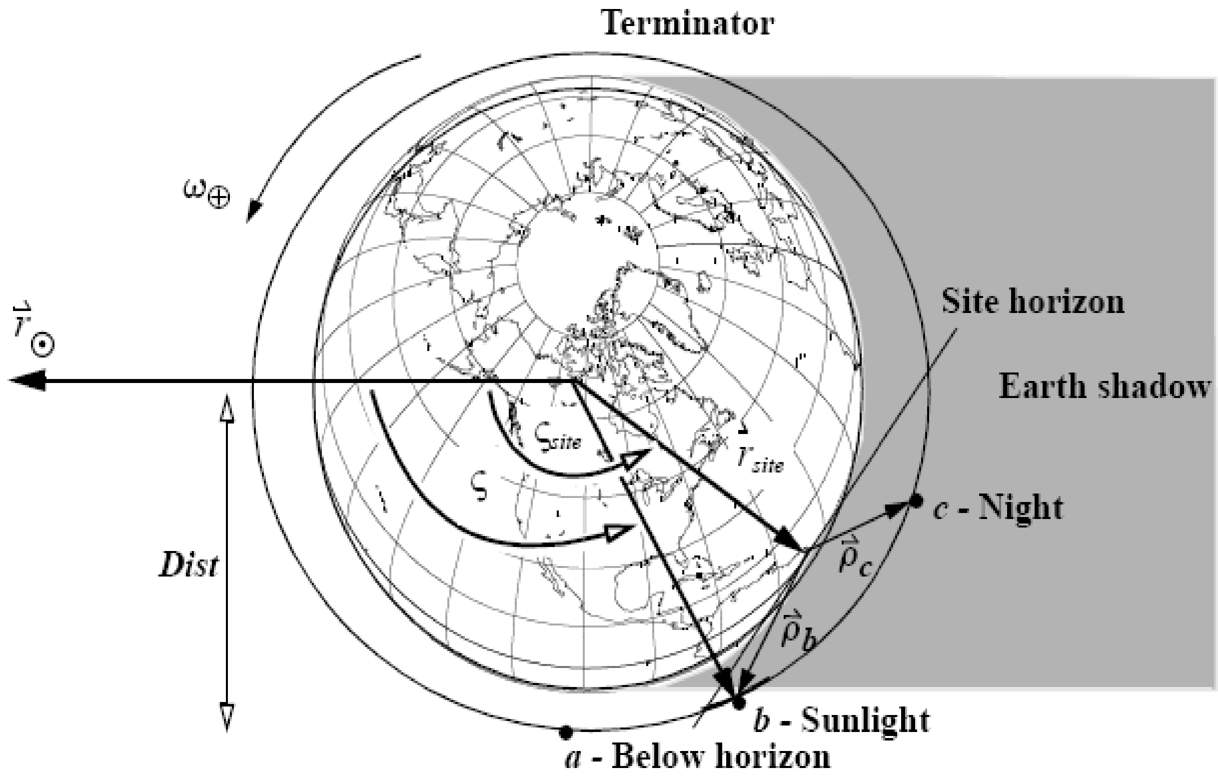
Applications

- How do we put all this together and accomplish our original goal?
  - Many analyses possible
    - Prediction
      - Satellite look angles (Our original question)
- Behind the scenes
  - Time of observations
  - Coordinate systems throughout
  - Orbit determination of observations to obtain a state vector
  - Propagation to form an ephemeris
  - Calculations for Sun and Satellite to determine visibility
  - ...
  - And several other smaller details!

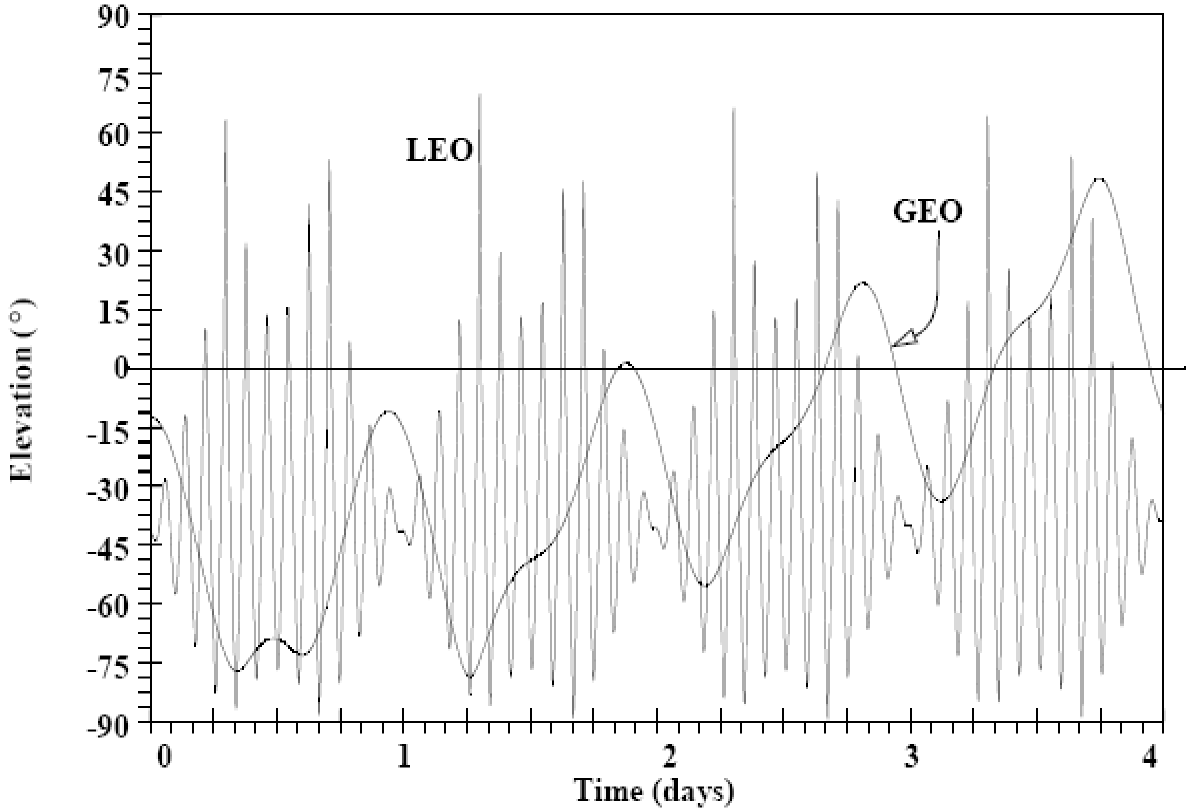
Satellite Orbital Characteristics



Predicting Satellite Look Angles



Rise Set Characteristics



Finding the Site Information (1)

- Approximate formulation
  - Non-rigorous ECEF
  - Don't account for sidereal/solar time differences

$$\vec{\rho}_{\sim ECI} = [ROT3(-\theta_{LST})][ROT2(-90^\circ - \phi_{gd})]\vec{\rho}_{SEZ}$$

$$\dot{\vec{\rho}}_{\sim ECI} = [ROT3(-\theta_{LST})][ROT2(-90^\circ - \phi_{gd})]\dot{\vec{\rho}}_{SEZ}$$

$$\vec{r}_{\sim ECI} = \vec{\rho}_{\sim ECI} + \vec{r}_{SiteECI}$$

$$\vec{v}_{\sim ECI} = \dot{\vec{\rho}}_{\sim ECI} + \vec{\omega}_{\oplus} \times \vec{r}_{\sim ECI}$$

Finding the Site Information (2)

- Rigorous formulation (STK approach)
  - Precise ECEF
  - Account for sidereal/solar time differences

$$\vec{\rho}_{ECEF} = [ROT3\boxed{(-\lambda)}][ROT2(-90° - \phi_{gd})]\vec{\rho}_{SEZ}$$

$$\dot{\vec{\rho}}_{ECEF} = [ROT3\boxed{(-\lambda)}][ROT2(-90° - \phi_{gd})]\dot{\vec{\rho}}_{SEZ}$$

$$\boxed{\begin{aligned}
&\vec{r}_{ECEF} = \vec{\rho}_{ECEF} + \vec{r}_{SiteECEF}\\
&\vec{v}_{ECEF} = \dot{\vec{\rho}}_{ECEF}\\
&(yr, mon, day, UTC, \Delta UT1, \Delta AT) \Rightarrow (UT1, TAI, TT, T_{UT1}, T_{TT})\\
&[PREC] = ROT3(-z)ROT2(\Theta)ROT3(-\zeta)\\
&[NUT] = ROT1(-\varepsilon)ROT3(-\Delta\Psi)ROT1(\bar{\varepsilon})\\
&[ST] = ROT3(\theta_{AST})\\
&[PM] = ROT2(-x_p)ROT1(-y_p)\\
&\vec{r}_{ECI} = [PREC]^T[NUT]^T[ST]^T[PM]^T\,\vec{r}_{ECEF}\\
&\vec{v}_{ECI} = [PREC]^T[NUT]^T[ST]^T\left\{[PM]^T\,\vec{v}_{ECEF} + \vec{\omega}_{\oplus} \times \vec{r}_{PEF}\right\}
\end{aligned}}$$

Differences from Approximate

Results

- Rigorous approach
  - Position (ECI)
    - -5505.504883 km
    - 56.449170
    - 3821.871726
- Simplified approach
  - Position (~ECI)
    - -5503.79562 km
    - 62.28191
    - 3824.24480
- Difference
  - 6.52 km
- Perhaps this is acceptable?

Impact

- Applying textbook solutions to real-world problems will give the wrong answers
  - Assumptions add up
  - Examples:
    - Communicating with a satellite using Laser comm
      - At orbital velocity, 2 sec is nearly 14 km
        - » Will your signal be able to locate and receive?
    - Will you pass System Acceptance Testing?

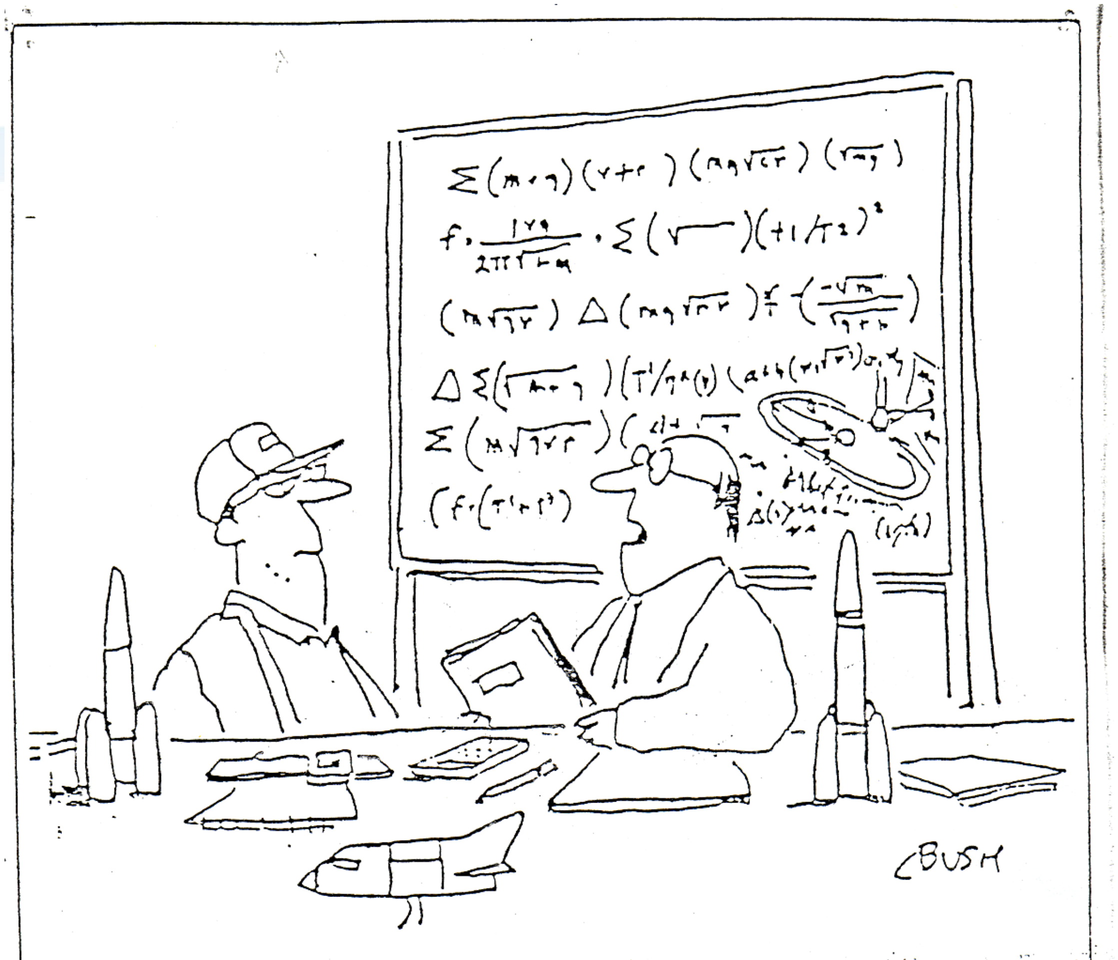
A word of Caution …

- Fundamentals vs Applications
  - Undergraduate vs Graduate
  - Classroom vs Operational
  - Attention to detail important
  - Nomenclature is important

Resources

- Book
  - Microcosm
    - Pam is here!
- http://www.celestrak.com/software/vallado-sw.asp
  - TLE data
  - EOP and Space Weather Data
  - Code
    - SGP4
    - Other
  - Errata
    - Not all updated but most are
  - Solutions
    - Not complete – ask ☺

Questions??



I was wrong, Cliff, you DO have to be a rocket scientist to figure this out!